ALTERNATING-CURRENT MACHINES:

BEING THE SECOND VOLUME OF

DYNAMO ELECTRIC MACHINERY;

ITS CONSTRUCTION, DESIGN, AND OPERATION

BY

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PREFACE.

THIS book, like its companion volume on Direct Current Machines, is primarily intended as a text-book for use in technical educational institutions. It is hoped and believed that it will also be of use to those electrical, civil, mechanical, and hydraulic engineers who are not perfectly familiar with the subject of Alternating Currents, but whose work leads them into this field. It is furthermore intended for use by those who are earnestly studying the subject by themselves, and who have previously acquired some proficiency in mathematics.

There are several methods of treatment of alternating-current problems. Any point is susceptible of demonstration by each of the methods. The use of all methods in connection with every point leads to complexity, and is undesirable in a book of this character. In each case that method has been chosen which was deemed clearest and most concise. No use has been made of the method of complex imaginary numbers.

A thorough understanding of what takes place in an alternating-current circuit is not to be easily acquired. It is believed, however, that one who has mastered the first four chapters of this book will be able to solve any practical problem concerning the relations which exist between power, electro-motive forces, currents, and their phases in

series or multiple alternating-current circuits containing resistance, capacity, and inductance.

The next four chapters are devoted to the construction, principle of operation, and behavior of the various types of alternating-current machines. Only American machines have been considered.

A large amount of alternating-current apparatus is used in connection with plants for the long-distance transmission of power. This subject is treated in the ninth chapter. The last chapter gives directions for making a variety of tests on alternating-current circuits and apparatus.

No apology is necessary for the introduction of cuts and material supplied by the various manufacturing companies. The information and ability of their engineers, and the taste and skill of their artists, are unsurpassed, and the information supplied by them is not available from other sources. For their courteous favors thanks is hereby given.

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ALTERNATING-CURRENT MACHINES.

CHAPTER I.

PROPERTIES OF ALTERNATING CURRENTS.

1. **Definition of an Alternating Current.** — An alternating current of electricity is a current which changes its direction of flow at regularly recurring intervals. Between these intervals the value of the current may vary in any way. In usual practice, the value varies with some regularity from zero to a maximum, and decreases with the same regularity to zero, then to an equal maximum in the other direction, and finally to zero again. In practice, too, the intervals of current flow are very short, ranging from $\frac{1}{50}$ to $\frac{1}{240}$ second.

2. **Frequency.** — When, as stated above, a current has passed from zero to a maximum in one direction, to zero, to a maximum in the other direction, and finally to zero again, it is said to have completed one *cycle*. That is to say, it has returned to the condition in which it was first considered, both as to value and as to direction, and is prepared to repeat the process described, making a second cycle. It should be noted that it takes two *alternations* to make one *cycle*. The tilde ($\sim$) is frequently used to denote cycles.

The term *frequency* is applied to the number of cycles completed in a unit time, i.e., in one second. Occasionally the word *alternations* is used, in which case, unless otherwise specified, the number of alternations per minute is meant. Thus the same current is spoken of as having a frequency of 25, or as having 3000 alternations. The use of the word alternations is condemned by good practice. In algebraic notation the letter f usually stands for the frequency.

The frequency of a commercial alternating current depends upon the work expected of it. For power a low frequency is desirable, particularly for converters. The great Niagara power plant uses a frequency of 25. Lamps, however, are operated satisfactorily only on frequencies of 50 or more. Early machines had higher frequencies, — 125 and 133 (16,000 alternations) being usual, — but these are almost entirely abandoned because of their increased losses and their unadaptability to the operation of motors and similar apparatus.

In the Report of the Committee on Standardization of the American Institute of Electrical Engineers is the following: "In alternating-current circuits, the following approximate frequencies are recommended as desirable:

25 or 30 40 60 120

"These frequencies are already in extensive use, and it is deemed advisable to adhere to them as closely as possible."

The frequency of an alternating current is always that of the *E.M.F.* producing it. To find the frequency of the pressure or the current produced by any alternating-cur-

rent generator, if V be the number of revolutions per minute, and p be the number of pairs of poles, then

$$f = p\frac{V}{60}.$$

3. **Wave-shape.** — If, in an alternating current, the instantaneous values of current be taken as ordinates, and time be the abscissæ, a curve, as in Fig. 1, may be developed. The length of the abscissa for one complete cycle is $\frac{1}{f}$ seconds.

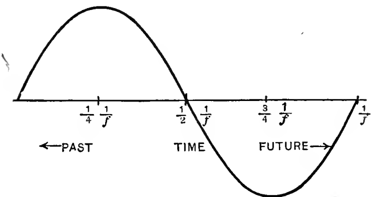


Fig. 1.

Imagine a small cylinder, Fig. 2, carried on one end of a wire, and rotated uniformly about the other end in a vertical plane. Imagine a horizontal beam of parallel rays of light to be parallel to the plane of rotation, and to cast a shadow of the cylinder on

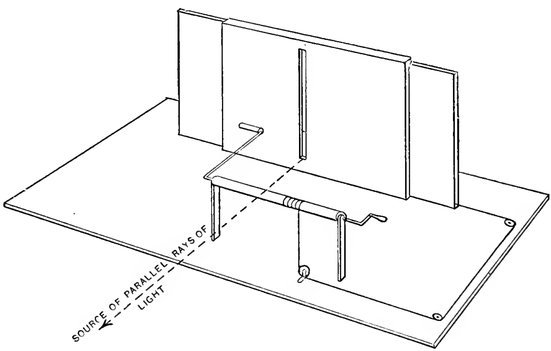


Fig. 2.

a plane screen perpendicular to the rays. The shadow will move up and down, passing from the top of its travel to the bottom in a half revolution, and from the bottom

back to the top in another half revolution with a perfect harmonic motion. Now imagine the screen to be moved horizontally in its own plane with a uniform motion, and the positions of the shadow suitably recorded on it, — as on sensitized paper or on a photographic film, a slotted screen protecting all but the desired portion from exposure. Then the trace of the shadow will be as in Fig. 3. The abscissae of this curve may be taken as time, as in the preceding curve, the abscissa of one complete cycle being the time in seconds of one revolution. Or, with equal relevancy, the abscissæ may be expressed in degrees. Consider the cylinder to be in a zero position when the radius to which it is attached is horizontal. Then the abscissa of any point is the angle which must be turned through in order that the cylinder may cast its shadow at that point. In this case the abscissa of a complete cycle will be 360°, or 2π. Consideration of the manner in which the curve has been formed shows that the ordinate of any point is proportional to the sine of the abscissa of that point, expressed in degrees. Hence this is called a *sinusoid* or *sine curve*.

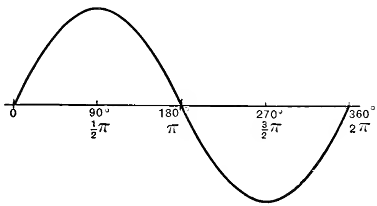


Fig. 3.

If the maximum ordinate of this curve be taken as E_m, and time be considered to commence at the beginning of any cycle, then the ordinate E' at any time t seconds later will be

$$E' = E_m \sin 2\,\pi f t,$$

which is equivalent to neglecting all those intervals of time corresponding to whole cycles, and considering only

the time elapsed since the end of the last completed cycle.

As a numerical example: In an alternating-current circuit of 45 ~ and a maximum voltage of 100, what will be the pressure at $2\frac{1}{8}$ seconds after the beginning of a cycle?

$$E' = 100 \sin (2\,\pi \times 45 \times 2.125)$$

$$\frac{E'}{100} = \sin 191.25\,\pi = \sin 1.25\,\pi = -\frac{1}{\sqrt{2}},$$

whence

$$E' = -70.7 \text{ volts.}$$

Since the ordinates of the curve may represent either current or pressure, the expression

$$I' = I_m \sin 2\pi ft$$

is equally true.

The ideal pressure curve from an alternator is sinusoidal. Commercial alternators, however, do not generate true sinusoidal pressures. But the sine curve can be treated with relative simplicity, and the curves of practice approximate so closely to the sine form, that mathematical deductions based on sine curves can with propriety be applied to those of practice. Two of these actual curves are shown in Fig. 4.

The shape of the pressure curve is affected by irregular distribution of the magnetic flux. Also uneven angular velocity of the generator will distort the wave-shape, making it, relative to the true curve, lower in the slow spots and higher in the fast ones. Again, the magnetic reluctance of the armature may vary in different angular positions, particularly if the inductors are laid in a few large slots. This would cause a periodic variation in the

reluctance of the whole magnetic circuit and a corresponding pulsation of the total magnetic flux. All these influences operate at open circuit as well as under load.

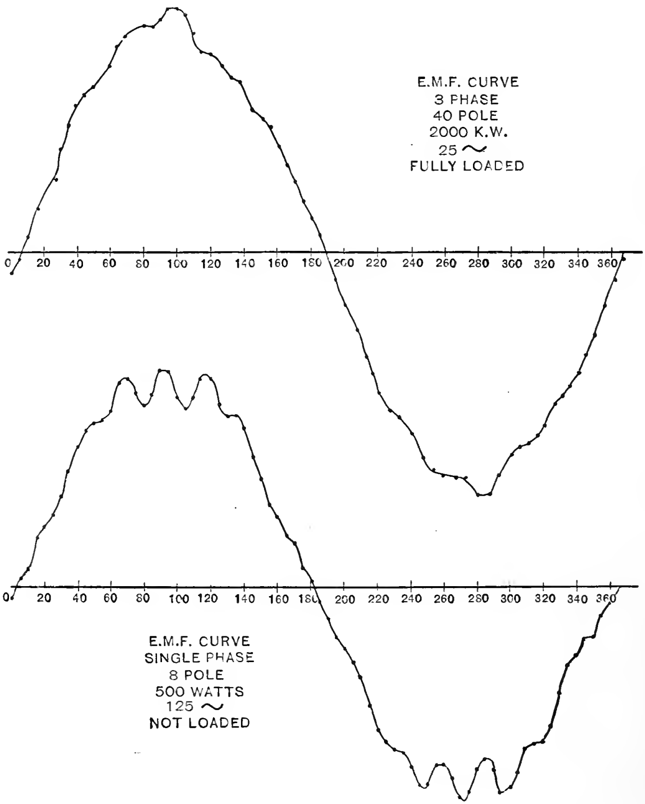


Fig. 4.

There are two other causes which act to distort the wave-shape only when under load. For any separately excited generator, a change in the resistance or apparent resistance of the external circuit will cause a change in the

terminal voltage of the machine. As is explained later, the apparent resistance (impedance) of a circuit to alternating currents depends upon the permeability of the iron adjacent to the circuit. Permeability changes with magnetization. Now, because an alternating current is flowing, the magnetization changes with the changing values of current. This, by varying the permeability, sets up a pulsation in the impedance and affects the terminal voltage of the machine, periodically distorting the wave of pressure from the true sine.

There are cases of synchronously pulsating resistances. The most common is that of the alternating arc. With the same arc the apparent resistance of the arc varies inversely as the current. So when operated by alternating currents, the resistance of a circuit of arc lamps varies synchronously, and distorts the pressure wave-shape in a manner analogous to the above.

Summing up, the wave-shape of pressure may be distorted: *At open circuit as well as under load;* by lack of uniformity of magnetic distribution, by pulsating of magnetic field, by variation in angular velocity of armature; and *under load only;* by pulsation of impedance, by pulsation of resistance. And the effects of any or all may be superimposed.

4. Effective Values of E.M.F. and of Current. — One ampere of alternating current is a current of such instantaneous values as to have the same heating effect in a conductor as one ampere of direct current. This somewhat arbitrary definition probably arose from the fact that alternating currents were first commercially employed in lighting circuits, where their utility was measured by the heat

they produced in the filaments; and further from the fact that the only means then at hand of measuring alternating currents were the hot-wire instruments and the electro-dynamometer, either of which gives the same indication for an ampere of direct current or for what is now called an ampere of alternating current.

The heat produced in a conductor carrying a current is proportional to the square of the current. In an alternating current, whose instantaneous current values vary, the instantaneous rate of heating is not proportional to the instantaneous value, nor yet to the square of the average of the current values, but to the square of the instantaneous current value. And so the average heating effect is proportional to the mean of the squares of the instantaneous currents.

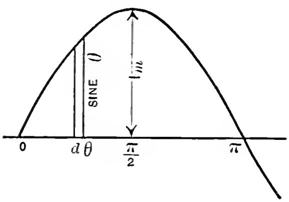


Fig. 5.

The *average* current of a sinusoidal wave of alternating current, whose maximum value is I_m, is equal to the area of one lobe of the curve, Fig. 5, divided by its base line π. Thus

$$I_{av} = \frac{\int_0^\pi I_m \sin\theta d\theta}{\pi} = \frac{I_m}{\pi}\left[-\cos\theta\right]_0^\pi = \frac{2}{\pi} I_m.$$

But the heating value of such a current varies, as

$$I^2 = \frac{\int_0^\pi I_m^2 \sin^2\theta d\theta}{\pi} = \frac{I_m^2}{\pi}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^\pi = \frac{1}{2} I_m^2.$$

The square root of this quantity is called the *effective* value of the current, $I = \frac{I_m}{\sqrt{2}}$. This has the same heating

effect as a direct current I, and the effective values are always referred to unless expressly stated otherwise. Alternating-current ammeters are designed to read in effective amperes.

Since current is dependent upon the pressure, the resistance or apparent resistance of a circuit remaining constant, it is obvious that if $I = \frac{I_m}{\sqrt{2}}$ then does also $E = \frac{E_m}{\sqrt{2}}$. Likewise if average $I = \frac{2}{\pi} I_m$ then does also average $E = \frac{2}{\pi} E_m$. Or these may be demonstrated in a manner analogous to the above.

The maximum value of pressure is frequently referred to in designing alternator armatures, and in calculating dielectric strength of insulation. There have arisen various ways of indicating that *effective* values are meant, for instance, the expressions, sq. root of mean sq., $\sqrt{\overline{e^2}}$, $\sqrt{\text{mean square}}$. In England the initials R.M.S. are frequently used for root mean square.

The ratio $\dfrac{\text{Effective } E.M.F.}{\text{Average } E.M.F.}$ is called the *form-factor*, since its value depends upon the shape of the pressure wave. For the curve Fig. 6, the form-factor is unity. As a curve becomes more peaked, its form-factor increases, due to the superior weight of the squares of the longer ordinates.

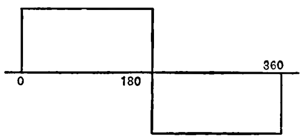


Fig. 6.

In the sinusoid the values found above give

$$\text{Form-factor} = \frac{\frac{1}{\sqrt{2}} E_m}{\frac{2}{\pi} E_m} = 1.11.$$

Probably no alternators give sine waves, but they approach it so nearly that the value 1.11 can be used in calculation without sensible error.

5. **Phase.** — The curves of the pressure and the current in a circuit can be plotted together, with their respective ordinates and common abscissæ, as in Fig. 7. In some cases the zero and the maximum values of the current curve will occur at the same abscissæ as do those values of the pressure curve, as in Fig. 7. In such a case the current is said to be *in phase* with the pressure.

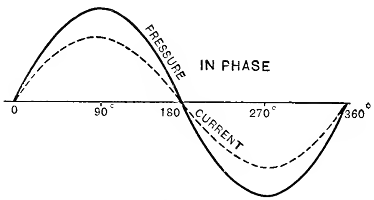


Fig. 7.

In other cases the current will reach a maximum or a zero value at a time later than the corresponding values of the pressure, and since the abscissæ are indifferently time or degrees, the condition is represented in Fig. 8. In such a case, the current is said to be *out of phase* with, and to *lag* behind the pressure. In still other cases the curves are placed as in Fig. 9, and the current and pressure are again *out of phase*, but the current is said to *lead* the pressure.

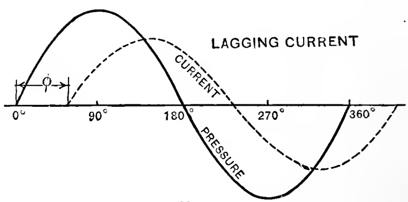


Fig. 8.

The distance between the zero ordinate of one sine curve and the corresponding zero ordinate of another, may be measured in degrees, and is called the *angular displacement* or *phase difference*. This *angle* of *lag* or of *lead* is usually represented by ϕ. When one

curve has its zero ordinate coincident with the maximum ordinate of the other, as in Fig. 10, there is a displacement of a quarter cycle ($\phi = 90°$), and the curves are said to be at right angles. This term owes its origin to the fact that the radii whose projections will trace these curves, as in § 3, are at right angles to each other.

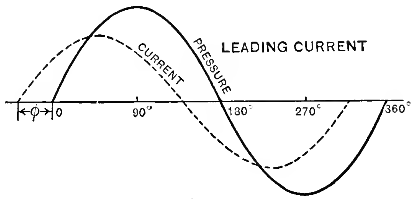


Fig. 9.

If the zero ordinates of the two curves coincide, but the positive maximum of one coincides with the negative maximum of the other, as in Fig. 11, then $\phi = 180°$, and the curves are in *opposite phase.*

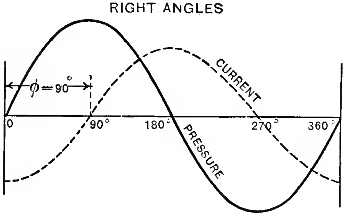


Fig. 10.

An alternator arranged to give a single pressure wave to a two-wire circuit is said to be a *single phaser,* and the current in the circuit a *single-phase current.* Some machines are arranged to give pressure to two distinct circuits — each of which, considered alone, is a single-phase circuit — but the time of maximum pressure in one is the time of zero pressure in the other, so that simultaneous pressure curves from the two circuits take the form of Fig. 12. Such is said to be a *two-phase* or *quarter-phase*

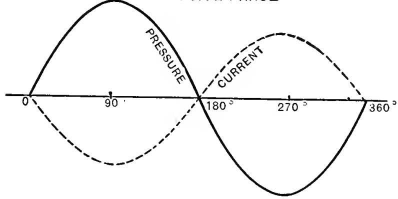


Fig. 11.

system, and the generator is a *two-phaser*. A *three-phase* system theoretically has three circuits of two wires each. The maximum positive pressure on any circuit is displaced from that of either of the other circuits by 120°. As the algebraic sum of the currents in all these circuits (if balanced) is at every instant equal to zero, the three return wires, one on each circuit, may be dispensed with, leaving but three wires. The three simultaneous curves of *E.M.F.* are shown in Fig. 13. The term *polyphase* applies to any system of two or more phases. An n-phase system has n circuits and n pressures with successive phase differences of $\frac{360}{n}$ degrees.

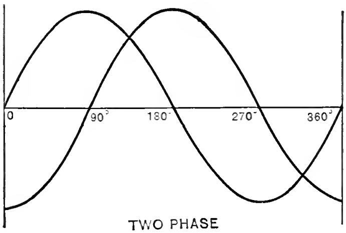


Fig. 12.

6. Power in Alternating-Current Circuits. — With a direct-current circuit, the power in the circuit is equal to the product of the pressure in volts by the current strength in amperes. In an alternating-current circuit, the *instantaneous* power is the product of the instantaneous values of current strength and pressure. If the current and pressure are out of phase there will be some instants when the pressure will have a positive value and the current a negative value or *vice versa*. At such times the instantaneous power will be a negative quantity, i.e.,

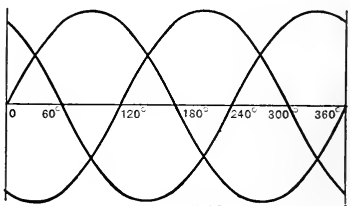


Fig. 13.

power is being returned to the generator by the disappearing magnetic field which had been previously produced by the current. This condition is shown in Fig. 14, where the power curve has for its ordinates the product of the corresponding ordinates of pressure and current. These are reduced by multiplying by a constant so as to make them of convenient size. The circuit, therefore, receives power from the generator and gives power back again in alternating pulsations having twice the frequency of the generator. It is clear that the relative magnitudes of the negative and positive lobes of the power curve will vary for different values of ϕ, even though the original curves maintain the same size and shape. So it follows that the power in an alternating-current circuit is not merely a function of E and I, as in direct-current circuits, but is a function of E, I, and ϕ, and the relation is deduced as follows:—

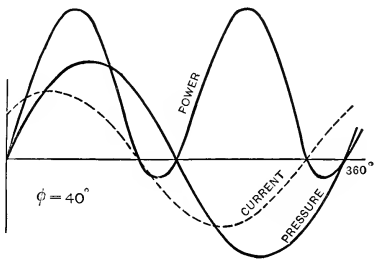


Fig. 14.

Let the accent ($'$) denote instantaneous values. If the current lag by the angle ϕ, then from § 3,

$$E' = E_m \sin \alpha,$$

where, for convenience,

$$\alpha = 2\,\pi f t,$$

and

$$I' = I_m \sin (\alpha - \phi).$$

Remembering that

$$E = \frac{E_m}{\sqrt{2}}, \text{ and } I = \frac{I_m}{\sqrt{2}} \text{ (§ 4) the instantaneous power,}$$

$$P' = E'\,I' = 2EI \sin \alpha \sin (\alpha - \phi).$$

But $$\sin(\alpha - \phi) = \sin\alpha\cos\phi - \cos\alpha\sin\phi,$$

so $$P' = 2\,EI(\sin^2\alpha\cos\phi - \sin\alpha\cos\alpha\sin\phi).$$

Remembering that ϕ is a constant, the average power over 180°,

$$P = \frac{2\,EI\cos\phi}{\pi}\int_0^\pi \sin^2\alpha d\alpha - \frac{2\,EI\sin\phi}{\pi}\int_0^\pi \sin\alpha\cos\alpha d\alpha$$

$$= \frac{2\,EI\cos\phi}{\pi}\left[\frac{1}{2}\alpha - \frac{1}{4}\sin 2\alpha\right]_0^\pi - \frac{2\,EI\sin\phi}{\pi}\left[\frac{1}{2}\sin^2\alpha\right]_0^\pi.$$

$$P = EI\cos\phi.$$

Should the current *lead* the pressure by $\phi°$, then the leading equation would be

$$P' = 2\,EI\sin\alpha\sin(\alpha + \phi),$$

which gives the same expression,

$$P = EI\cos\phi,$$

which is the general expression for power in an alternating-current circuit.

Since, to get the true power in the circuit, the apparent power, or volt-amperes, must be multiplied by cos ϕ, this quantity is called the *power factor* of the circuit. If the pressure and current are in phase, $\phi = 0°$, and the power factor is unity.

CHAPTER II.

SELF-INDUCTION.

7. **Self-Inductance.** — The subject of inductance was briefly treated of in § 15, vol. i., of this work; but, since it is an essential part of alternating-current phenomena, it will be discussed more fully in this chapter. When lines of force are cut by a conductor an *E.M.F.* is generated in that conductor (§ 13, vol. i.). A conductor carrying current is encircled by lines of force. When the current is first started in such a conductor, these lines of force must be established. In establishing itself, each line is considered as having cut the conductor, or, what is equivalent, been cut by the conductor. This notion of lines of force is a convenient fiction, designed to render an understanding of the subject more easy. To account for the *E.M.F.* of self-induction, the encircling lines must be considered as cutting the conductor which carries the current that establishes them, during their establishment. It may be considered that they start from the axis of the conductor at the moment of starting the current in the circuit; that they grow in diameter while the current is increasing; that they shrink in diameter when the current is decreasing; and that all their diameters reduce to zero upon stopping the current. At any given current strength the conductor is surrounded by many circular lines, the circles having various diameters. Upon decreasing the strength those of

smaller diameter cut the conductor and disappear into a point on the axis of the conductor previous to the cutting by those of larger diameter. The number of lines accompanying a large current is greater than the number accompanying a smaller current.

The *E.M.F.* of self-induction is always a *counter E.M.F.* By this is meant that its direction is such as to tend to prevent the change of current which causes it. When the current is started the self-induced pressure tends to oppose the flow of the current and prevents its reaching its full value immediately. When the circuit is interrupted the *E.M.F.* of self-induction tends to keep the current flowing in the same direction that it had originally.

8. Unit of Self-Inductance. — The *self-inductance*, or the *coefficient of self-induction* of a circuit is generally represented by L or l, and is that constant by which the time rate of change of the current in a circuit must be multiplied in order to give the *E.M.F.* induced in that circuit. Its absolute value is numerically equal to the number of lines of force linked with the circuit, per absolute unit of current in the circuit, as is shown below. By linkages, or number of lines linked with a circuit, is meant the sum of the number of lines surrounding each portion of the circuit. For instance, a coil of wire consisting of ten turns, and threaded completely through by twelve lines of force, is said to have 120 linkages.

The absolute unit of self-inductance is too small for ordinary purposes, and a practical unit, the *henry*, is used. This is 10^9 times as large as the C. G. S. or absolute unit.

The Paris electrical congress of 1900 adopted as the unit of magnetic flux the maxwell, and of flux density the

gauss. A maxwell is one line of force. A gauss is one line of force per square centimeter. If a core of an electromagnet has a transverse cross-section of 30 sq. cm., and is uniformly permeated with 60,000 lines of force, such a core may be said to have a flux of 60,000 maxwells and a flux density of 2000 gausses.

In § 13, vol. I., it has been shown that the pressure generated in a coil of wire when it is cut by lines of force is

$$e = -\frac{n\,d\Phi}{dt},$$

where n is the number of turns in a coil, and where e is measured in C. G. S. units, Φ in maxwells, and t in seconds. In a simple case of self-induction the maxwells set up are due solely to the current in the conductor. Now let K be a constant, dependent upon the permeability of the magnetic circuit, such that it represents the number of maxwells set up per unit current in the electric circuit; then, indicating instantaneous values by prime accents,

$$\Phi' = Ki',$$

and $$d\Phi = Kdi.$$

The *E.M.F.* of self-induction may then be written

$$e_s = - Kn\frac{di}{dt}. \checkmark$$

By the definition of the coefficient of self-induction, whose C. G. S. value is represented by l,

$$e_s = - l\frac{di}{dt}.$$

From the last two equations, it is seen that $l = Kn$. Kn is evidently the number of linkages per absolute unit current. The negative sign indicates that the pressure is counter *E.M.F.*

?

In practical units,

$$E_s = - L \frac{dI}{dt} \cdot$$

A circuit having an inductance of one henry will have a pressure of one volt induced in it by a uniform change of current of one ampere per second.

9. Practical Values of Inductances. — To give the student an idea of the values of self-inductance met with in practice, a number of examples are here cited.

A pair of copper line wires, say a telephone pole line, will have from two to four milhenrys (.002 to .004 henrys) per mile, according to the distance between them, the larger value being for the greater distance.

The secondary of an induction coil giving a 2″ spark has a resistance of about 6000 ohms and 50 henrys.

The secondary of a much larger coil has 30,000 ohms and about 2000 henrys.

A telephone call bell with about 75 ohms has 1.5 henrys.

A coil found very useful in illustrative and quantitive experiments in the alternating-current laboratory is of the following dimensions. It is wound on a pasteboard cylinder with wooden ends, making a spool 8.5 inches long and 2 inches internal diameter. This is wound to a depth of 1.5 inch with No. 16 B. and S. double cotton-covered copper wire, there being about 3000 turns in all. A bundle of iron wires, 16 inches long, fits loosely in the hole of the spool. The resistance of the coil is 10 ohms, and its inductance without the core is 0.2 henry. With the iron core in place and a current of about 0.2 ampere, the inductance is about 1.75 henrys. This coil is referred to again in § 11.

The inductance of a spool on the field frame of a generator is numerically

$$L = \frac{\Phi n}{10^8 I_f},$$

where Φ is the total flux from one pole, n the number of turns per spool, and I_f the field current of the machine. It is evident that the value of L may vary through a wide range with different machines.

10. Things Which Influence the Magnitude of L.— If all the conditions remain constant, save those under consideration, then the self-inductance of a coil will vary: directly as the square of the number of turns; directly as the linear dimension if the coil changes its size without changing its shape; and inversely as the reluctance of the magnetic circuit.

Any of the above relations is apparent from the following equations. The numerical value of the self-inductance is

$$l = n\frac{\Phi}{i}.$$

As shown in Chapter 2, vol. i.,

$$\Phi = \frac{M.M.F.}{\text{reluctance}} = \frac{4\,\pi n i}{\frac{c}{\mu A}},$$

where c is the mean length in centimeters of the magnetic circuit, A its mean cross-sectional area in square centimeters, and μ is permeability.

Then, if $\mathcal{R}$ stand for the reluctance,

$$l = \frac{n}{i}\cdot\frac{4\,\pi n i}{\frac{c}{\mu A}} = 4\,\pi n^2 \mu\frac{A}{c} = \frac{4\,\pi n^2}{\mathcal{R}},$$

which is independent of i.

If, as is generally the case, there is iron in the magnetic circuit, it is practically impossible to keep μ constant if any of the conditions are altered; and it is to be particularly noted, that with iron in the magnetic circuit, L is by no means independent of I.

11. Growth of Current in an Inductive Circuit. — If a constant $E.M.F.$ be applied to the terminals of a circuit having both resistance and inductance, the current does not instantly assume its full ultimate value, but logarithmically increases to that value.

At the instant of closing the circuit there is no current flowing. Let time be reckoned from this instant. At any subsequent instant, t seconds later, the impressed $E.M.F.$ may be considered as the sum of two parts, E_1 and E_r. The first, E_1, is that part which is opposed to, and just neutralizes, the $E.M.F.$ of self-induction, so that $E_1 = -E_s$;

but
$$E_s = -L\frac{dI}{dt},$$

so
$$E_1 = L\frac{dI}{dt}.$$

The second part, E_r, is that which is necessary to send current through the resistance of the circuit, according to Ohm's Law, so that

$$E_r = RI.$$

If the impressed $E.M.F.$

$$E = E_r + E_1 = RI + L\frac{dI}{dt},$$

then
$$(E - RI)\,dt = LdI,$$

and
$$dt = \frac{L}{E - RI}\,dI = -\frac{L}{R}\cdot\frac{RdI}{E - RI}.$$

Integrating from the initial conditions $t=0$, $I=0$ to any conditions $t=t$, $I=I'$,

$$t = -\frac{L}{R}[\log(E - RI') - \log E]$$

$$-\frac{Rt}{L} = \log\left(\frac{E - RI'}{E}\right),$$

and $$I' = \frac{E}{R} - \frac{E}{R}\epsilon^{-\frac{R}{L}t} = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right),$$

where ϵ is the base of the natural system of logarithms.

This equation shows that the rise of current in such a circuit is along a logarithmic curve, as stated, and that when t is of sufficient magnitude to render the term $\epsilon^{-\frac{R}{L}t}$ negligible the current will follow Ohm's Law, a condition that agrees with observed facts.

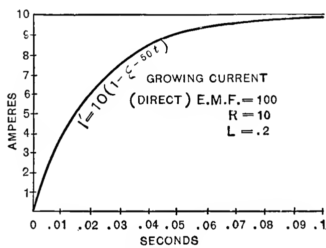


Fig. 15.

Fig. 15 shows the curve of growth of current in the coil referred to in § 9. The curve is calculated by the above formula for the conditions noted.

The ratio $\frac{L}{R}$ is called the *time constant* of the circuit, for the greater this ratio is, the longer it takes the current to obtain its full ultimate value.

12. Decay of Current in an Inductive Circuit. — If a current be flowing in a circuit containing inductance and resistance, and the supply of *E.M.F.* be discontinued, without, however, interrupting the continuity of the circuit, the current will not cease instantly, but the *E.M.F.* of

self-induction will keep it flowing for a time, with values decreasing according to a logarithmic law.

An expression for the value of this current at any time, t seconds after cutting off the source of impressed *E.M.F.*, may be obtained as in the preceding section. Let time be reckoned from the instant of interruption of the impressed *E.M.F.* The current at this instant may be represented by $\frac{E}{R}$, and is due solely to the *E.M.F.* of self-induction.

Therefore, from Ohm's Law,

$$E_s = RI = -L\frac{dI}{dt}.$$

$$\therefore\ dt = -\frac{L}{R}\frac{dI}{I}.$$

Integrating from the initial conditions $t = 0$, $I = \frac{E}{R}$, to the conditions, $t = t$, $I = I'$,

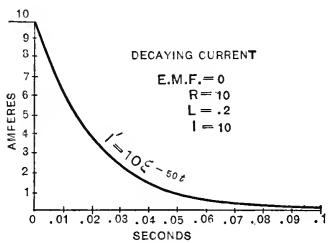


Fig. 16.

$$\int_0^t dt = -\frac{L}{R}\int_{\frac{E}{R}}^{I'}\frac{dI}{dt}$$

$$t = -\frac{L}{R}\log\frac{I'}{\frac{E}{R}},$$

and $$I' = \frac{E}{R}\epsilon^{-\frac{R}{L}t},$$

which is seen to be the term that had to be subtracted in the formula for growth of current. This shows clearly that while self-induction prevents the instantaneous attainment of the normal value of current, there is eventually no loss of energy, since what is subtracted from the growing current is giver back to the decaying current.

Fig. 16 is the curve of decay of current in the same cir-

cuit as was considered in Fig. 15. The ordinates of the one figure are seen to be complementary to those of the other.

13. Magnetic Energy of a Started Current. — If a current I is flowing under the pressure of E volts, the power expenditure is EI watts, and the work performed in the interval of time dt is

$$dW = EI\,dt.$$

But in a coil of n turns, the *E.M.F.* induced by a change of linkages is

$$E = -\frac{n d\Phi}{10^8\,dt}.$$

Substituting,

$$dW = -\frac{nI}{10^8}\,d\Phi.$$

If the circuit have a constant permeability,

$$n d\Phi = l di = 10^8 L dI,$$

so

$$dW = -LIdI.$$

Integrating through the full range, from 0 to W and from 0 to I,

$$\int_0^W dW = -L\int_0^I I\,dI,$$

$$W = -\tfrac{1}{2}LI^2.$$

Which is an expression for the work done *upon* the magnetic field in starting the current. When the current is stopped the work is done *by* the field, and the energy is returned to the circuit.

This formula assumes the value of L to be constant during the rise and fall of the current. If there be iron in the magnetic circuit the relation $n d\Phi = l di$ becomes $n d\Phi$

$= l'di$, l' being also a variable; but an average of the values of l' throughout the range may be called l, and the formula for energy stored in the field holds true.

Since iron has always a hysteretic loss, some of the energy is consumed, and the work given back at the disappearance of the field is less than that used to establish the field by the amount consumed in hysteresis.

14. Current Produced by a Harmonic E.M.F. in a Circuit Having Resistance and Inductance. — Given a circuit of resistance R and inductance L upon which is impressed a harmonic $E.M.F.$ E of frequency f, to find the current I in that circuit.

Represent by ω the quantity $2\pi f$.

At any instant of time, t, let the instantaneous value of the current be I'.

To maintain this current requires an $E.M.F.$ whose value at this instant is $I'R$. Represent this by E'_r.

From § 3, in a harmonic current,

$$I' = I_m \sin \omega t,$$

hence,

$$E_r' = RI_m \sin \omega t.$$

Evidently E_r' has its maximum value $RI_m = E_{rm}$ at $\omega t = 90°$. or $270°$, and its effective value is $E_r = RI$.

The counter $E.M.F.$ of self-induction at the same instant of time, t, is

$$E_s' = - L \frac{dI'}{dt}.$$

But as before,

$$I' = I_m \sin \omega t.$$

so

$$dI' = \omega I_m \cos \omega t \, dt.$$

and

$$E_s' = - \omega L I_m \cos \omega t.$$

Evidently E_s' has a maximum value of $-\omega L I_m = E_{sm}$ at $\omega t = 0°$ or $180°$, and its effective value

$$E_s = -\omega L I.$$

It is clear that the impressed *E.M.F.* must be of such a value as to neutralize E_s and also supply E_r. But these two pressures cannot be simply added, since the maximum value of one occurs at the zero value of the other; that is, they are at right angles to each other, as defined in § 5. Reference to Fig. 17 will make it clear that combining these at right angles will give as a resultant the pressure $\sqrt{E_r^2 + E_s^2}$; and it is this pressure that the impressed *E.M.F.* E must equal and oppose. So

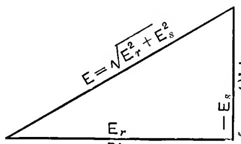


Fig. 17.

$$E = \sqrt{(IR)^2 + (\omega L I)^2},$$

from which

$$I = \frac{E}{\sqrt{R^2 + \omega^2 L^2}}.$$

This is a formula which must be used in place of Ohm's Law when treating inductive circuits carrying harmonic currents. It is evident that, if the inductance or the frequency be negligibly small (direct current has $f = 0$), the formula reduces to Ohm's Law; but for any sensible values of ω and L the current in the circuit will be less than that called for by Ohm's Law.

The expression $\sqrt{R^2 + \omega^2 L^2}$ is called the *impedance* of the circuit, and also the *apparent resistance*. The term R is of course called *resistance*, while the term ωL, which is $2\pi f L$, is called the *reactance*. Both are measured in ohms.

The effective value of the counter *E.M.F.* of self-induc-

tion can be determined as follows, without employing the calculus; that it must be combined at right angles with RI is not directly evident. Disregarding the direction of flow, an alternating current i reaches a maximum value i_m, $2f$ times per second. The maximum number of lines of force linked with the circuit on each of these occasions is li_m. The interval of time, from when the current is zero with no linkages, to when the current is a maximum with li_m linkages, is $\frac{1}{4f}$ second. The average rate of cutting lines, then, is $\frac{li_m}{\frac{1}{4f}}$, and is equal to the average $E.M.F.$ of self-induction during the interval. It has the same value during succeeding equal intervals; i.e.,

$$e_{s\,av} = -\frac{li_m}{\frac{1}{4f}} = -4fli_m.$$

The effective value is (§ 4) therefore,

$$e_s = -2\pi fli = \omega li,$$

and in practical units,

$$E_s = -2\pi fLI.$$

Since the squares of the quantities R, L, and ω enter into the expression for the impedance, if one, say R, is moderately small when compared with L or ω, its square will be negligibly small when compared with L^2 or ω^2. The frequency, because it is a part of ω, may be a considerable factor in determining the impedance of a circuit.

Having recourse once again to the harmonic shadowgraph described in § 3, the phase relation between impressed $E.M.F.$ and current may be made plain. It has already been shown that E_r and E_s are at right angles to

each other. Since the pressure E_r is the part of the impressed $E.M.F.$ which sends the current, the current must be in phase with it. Therefore there is always a phase displacement of 90° between I and E_s. This relation is also evident from a consideration of the fact that when I reaches its maximum value it has, for the instant, no rate of change; hence the flux, which is in phase with the current, is not changing, and consequently the $E.M.F.$ of self-induction must be, for the instant, zero. That is, I is maximum when E_s is zero, which means a displacement of 90°.

In Fig. 18 the triangle of $E.M.F.$'s of Fig. 17 is altered to the corresponding parallelogram of $E.M.F.$'s, and the maximum values substituted for the effective. If now the parallelogram revolve about the center o, the traces of the harmonic shadows of the extremities of E_m, E_{rm} and E_{sm} will develop as shown.

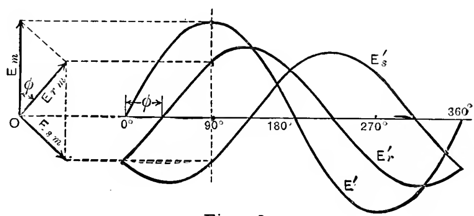


Fig. 18.

It is evident that the curve E_r' — and so also the curve of current — lags behind the curve E' by the angle ϕ. It is clear that the magnitude of ϕ depends upon the relative values of L and R in the circuit, the exact relation being derived from the triangle of forces.

$$\tan\phi = \frac{E_s}{E_r} = \frac{\omega LI}{RI} = \frac{\omega L}{R} = \frac{2\,\pi fL}{R}.$$

Furthermore

$$\cos\phi = \frac{E_r}{E},$$

that is, the cosine of the angle of lag is equal to the ratio

of the volts actually engaged in sending current to the volts impressed in the circuit, and this ratio is again equal to the power-factor as stated in § 6.

15. Choke Coils. — The term choke coil is applied to any device designed to utilize counter electromotive force of self-induction to cut down the flow of current in an alternating-current circuit. Disregarding losses by hysteresis, a choke coil does not absorb any power, except that which is due to the current passing through its resistance. It can therefore be more economically used than a rheostat which would perform the same functions.

These coils are often used on alternating-current circuits in such places as resistances are used on direct-current circuits. For instance, in the starting devices employed in connection with alternating-current motors, the counter *E.M.F.* of inductance is made to cut down the pressure applied at the motor terminals. The starter for direct-current motors employs resistance.

Since a lightning discharge is oscillatory in character and of enormous frequency, a coil which would offer a negligible impedance to an ordinary alternating current will offer a high impedance to a lightning discharge. This fact is recognized in the construction of lightning arresters. A choke coil of but few turns will offer so great an impedance to a lightning discharge that the high-tension, high-frequency current will find an easier path to the ground through an air gap suitably provided than through the machinery, and the latter is thus protected.

Choke coils are also used in connection with alternating-current incandescent lamps, to vary the current passing through them, and in consequence to vary the brilliancy.

CHAPTER III.

CAPACITY.

16. Condensers. — Any two conductors separated by a dielectric constitute a *condenser*. In practice the word is generally applied to a collection of thin sheets of metal separated by thin sheets of dielectric, every alternate metal plate being connected to one terminal of the instrument, the intervening plates to the other terminal. The Leyden jar is also a common form of condenser.

The office of a condenser is to store electrical energy by utilizing the principle of electrostatic induction. If a continuous *E.M.F.* be applied to the terminals of a condenser, a current will flow, large at first and gradually diminishing, till the plates of the condenser have been charged to an electrostatic difference of potential that equals and opposes the electrodynamic pressure applied. Then there is a balance of *E.M.F.*'s, and no current will flow if there be no leakage.

A frequent misconception as to the capacity of a condenser is that it is equal to the quantity of electricity it will hold. The quantity of electricity a given condenser will hold is directly proportional to the tension of the charge, and a consideration of this fact leads to the following definition :

The capacity of a condenser is numerically equal to the quantity of electricity with which it must be charged in

order to raise the potential difference between its terminals from zero to unity.

If the quantity and potential be measured in C. G. S. units, the capacity, c, will be in C. G. S. units. If practical units be employed, the capacity, c, is expressed in *farads*. The farad is the practical unit of capacity. A condenser whose potential is raised one volt by a charge of one coulomb has one farad capacity. The farad is 10^{-9} times the absolute unit, and even then is too large to conveniently express the magnitudes encountered in practice. The term microfarad ($\frac{1}{1,000,000}$ farad) is in most general use.

In electrostatics, both air and glass are used as dielectrics in condensers; but the mechanical difficulties of construction necessitate a low capacity per unit volume, and therefore render these substances impracticable in electrodynamic engineering. Mica, although it is expensive and difficult of manipulation, is generally used as the dielectric in standard condensers and in those which are intended to withstand high voltages. Many commercial condensers are made from sheets of tinfoil, alternating with slightly larger sheets of paraffined paper. Though not so good as mica, paraffin will make a good dielectric if properly treated. It is essential that all the moisture be expelled from the paraffin when employed in a condenser. If it is not, the water particles are alternately attracted and repelled by the changes of potential on the contiguous plates, till, by a purely mechanical action, a hole is worn completely through the dielectric, and the whole condenser rendered useless by short-circuit. Ordinary paper almost invariably contains small particles of metal, which become detached from the calendar rolls used in manufacture.

These occasion short-circuits even when the paper is doubled.

A distinctly different form of condenser is the electrolytic condenser. It consists of two electrodes dipping into an electrolyte, as, for instance, two carbon electrodes in zinc sulphate. A charge of electricity will deposit zinc upon one electrode and set up an *E.M.F.* of polarization. Such condensers should not be subjected to voltages in excess of their *E.M.F.* of polarization. Electrolytic condensers have about the same volume as other condensers of the same volt-ampere capacity.

The maximum voltage that may be applied to a condenser is limited by the dielectric strength of the material employed. If this limit be exceeded, the dielectric will be ruptured, which renders the condenser useless. The ohmic resistance of condenser dielectrics is not infinite. There is always a leakage from one charged plate to the other through the insulation and over its surface. Poor insulation may occasion a considerable loss, which appears as heat in the apparatus when in use. There is also a *dielectric hysteresis* which is analogous to magnetic hysteresis in iron. A dielectric with a high hysteretic constant may consume considerable power when in use, which will also appear as heat.

The capacity of a condenser may be calculated by using the following formula:—

$$C = .000225 \frac{An}{t} k,$$

where

C = capacity in microfarads,

A = area of dielectric between two conducting plates in square inches,

n = number of sheets of dielectric,
t = thickness of dielectric in mils,
k = specific inductive capacity of dielectric as obtained from the following table.

TABLE.

Glass	3 to 7
Ebonite	2.2 to 3
Gutta-percha	2.5
Paraffin	2 to 2.3
Shellac	2.75
Mica	6.6
Beeswax	1.8
Kerosene	2 to 2.5

17. Connection of Condensers in Parallel and in Series. — Condensers may be connected in parallel as in Fig. 19. If the capacities of the individual condensers be respectively C_1, C_2, C_3, etc., the capacity C of the combination will be

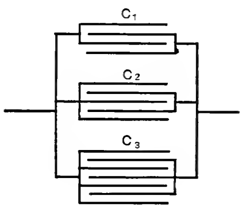


Fig. 19.

$$C = C_1 + C_2 + C_3 + \cdots.$$

The parallel arrangement of several condensers is equivalent to increasing the number of plates in one condenser. An increase in the number of plates results in an increase in the quantity of electricity necessary to raise the potential difference between the terminals of the condenser one volt; that is, an increase in the capacity results.

If the condensers be connected in series, as in Fig. 20, the capacity of the combination will be

$$C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots}.$$

For, if a quantity of positive electricity, Q, flow into the left side of C_1, it will induce and keep bound an equal negative quantity on the right side of C_1, and will repel an equal positive quantity. This last quantity will constitute the charge for the left side of C_2. The operation is repeated in the case of each of the condensers.

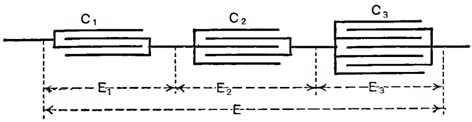


Fig. 20.

It is thus clear that the quantity of charge in each condenser is Q. The impressed *E.M.F.* must consist of the sum of the potential differences on the separate condensers. Let these differences be respectively E_1, E_2, E_3, etc. Then the impressed *E.M.F.*

$$E = E_1 + E_2 + E_3 + \cdot \cdot \cdot \cdot$$

But $\qquad E_1 = \frac{Q}{C_1}, \qquad E_2 = \frac{Q}{C_2}, \qquad E_3 = \frac{Q}{C_3},$ etc.,

and also, $\qquad E = \frac{Q}{C},$

therefore $\qquad \frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} = \frac{Q}{C_3} + \cdot \cdot \cdot \cdot$

or $\qquad C = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdot \cdot \cdot \cdot}.$

As an example, consider three condensers of respective capacities of 1, 2, and 5 microfarads. Since the factor to reduce to farads will appear on both sides of the equations, it may here be omitted. With the three in multiple (Fig. 19), the capacity of the combination will be

$$C = 1 + 2 + 5 = 8 \text{ mf.}$$

With the three in series (Fig. 20),

$$C = \frac{1}{\frac{1}{1} + \frac{1}{2} + \frac{1}{5}} = .588 \text{ mf.}$$

With the two smaller in parallel and in series with the larger (Fig. 21),

$$C = \frac{1}{\frac{1}{1+2} + \frac{1}{5}} = 1.875 \text{ mf.}$$

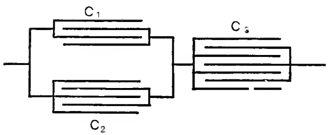


Fig. 21.

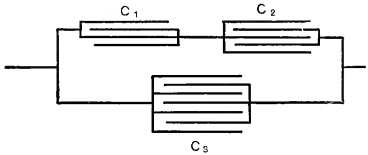


Fig. 22.

With the two smaller in series and in parallel with the larger (Fig. 22),

$$C = \frac{1}{\frac{1}{1} + \frac{1}{2}} + 5 = 5.666 \text{ mf.}$$

If with any condensers

$$C_1 = C_2 = C_3 = \cdot \cdot \cdot \cdot = C_n,$$

then, with n in multiple,

$$C = nC_1,$$

and with n in series,

$$C = \frac{1}{n} C_1.$$

It is interesting to note that the formulas for *capacities* in parallel and in series respectively are just the reverse of those for *resistances* in parallel and in series respectively.

18. Growth of Current in a Condensive Circuit.—The opposition to a flow of current which is caused by a con-

denser is quite different from that which is caused by a resistance. To be sure, there is some resistance in the leads and condenser plates, but this is generally so small as to be negligible. The practically infinite resistance of the condenser dielectric does not obstruct the current as an ordinary resistance is generally considered to do. The dielectric is the seat of a polarization *E.M.F.* which is developed by the condenser charge and which grows with it. It is a counter *E.M.F.*; and when it reaches a value equal to that of the impressed voltage, the charging current is forced to cease.

To find the current at any instant of time, t, in a circuit (Fig. 23) containing a resistance R and a capacity C, the constant impressed pressure E must be considered as consisting of two variable parts, one E_r, being active in sending current through the resistance, and the other part, E_c, being required to balance the potential of the condenser. Then at all times

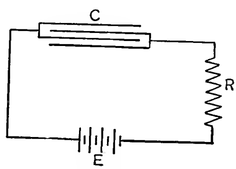


Fig. 23.

$$E = E_r' + E_c'.$$

Let time be reckoned from the instant the pressure E is applied; when, therefore, $t = 0$ and $I_0 = \frac{E}{R}$. Consider the current at any instant of time to be I'. Then if it flow for dt seconds it will cause dQ coulombs to traverse the circuit, and

$$I' = \frac{dQ'}{dt} \text{ or } dQ' = I' dt,$$

from which

$$Q' = \int I' dt.$$

By definition,
$$C = \frac{Q'}{E_c'},$$

therefore,
$$E_c' = \frac{Q'}{C} = \frac{\int I' dt}{C}.$$

And by Ohm's Law,
$$E_r' = I' R,$$

so at this instant of time
$$E = E_r' + E_c' = I'R + \frac{\int I' dt}{C},$$

whence
$$EC = RCI' + \int I' dt,$$

which upon differentiating, becomes
$$0 = RC dI' + I' dt.$$

Integrating
$$\int_0^t dt = -RC \int_{I_0}^{I'} \frac{dI'}{I'},$$
$$t = -RC\left[\log I' - \log \frac{E}{R}\right].$$

Solving for I',
$$I' = \frac{E}{R} \epsilon^{-\frac{t}{RC}},$$

which is the expression sought. Like the corresponding expression for an inductive circuit, it is logarithmic.

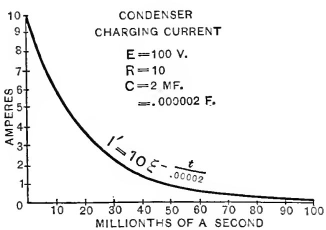


Fig. 24.

Fig. 24 is a curve showing the growth of current in a condenser for the conditions indicated.

19. Condensers in Alternating-Current Circuits — Hydraulic Analogy. — Imagine a circuit consisting of a pipe through which water is made to flow, first one way, then the other, by a piston oscillated pump-like in one section of it. The pipe circuit corresponds to an electric circuit, the pump to a generator of alternating *E.M.F.*, and the flow of water to a flow of alternating current. Further imagine one section of the pipe to be enlarged, and in it placed a transverse elastic diaphragm. This section corresponds to a condenser. Its capacity with a unit pressure of water on one side depends upon the area of the diaphragm, its thinness, and the elastic co-efficients of the material of which it is made. In a condenser the capacity depends upon the area of the dielectric under strain, its thinness, and the specific inductive capacity of the dielectric employed. As the water surges to and fro in the pipe, some work must be done upon the diaphragm, since it is not perfectly elastic. This loss corresponds to the loss in a condenser by dielectric hysteresis. The fact that the diaphragm is not absolutely impervious to water corresponds to the fact that a dielectric is not an absolute electric insulator. As the diaphragm may be burst by too great a hydrostatic pressure, so may the dielectric be ruptured by too great an elastic pressure.

20. Phase Relations. — To understand the relation between pressure and current in a condensive circuit, consider the above analogy. Imagine the diaphragm in its medial position, with equal volumes of water on either side of it, and the piston in the middle of its travel. This middle point corresponds to zero pressure. When the piston is completely depressed, there is a maximum negative

pressure, when completely elevated, a maximum positive pressure, if pressure and flow upward be considered in the positive direction. If the piston oscillate in its path with a regular motion, it is clear that the water will flow upward from the extreme lowest to the extreme highest position of the piston. That is, there will be flow in the positive direction from the maximum negative to the maximum positive values of pressure. The direction of flow is seen to remain unchanged while the piston passes through its middle position or the point of zero pressure.

Returning to electric phenomena, if a harmonic *E.M.F.* be impressed upon any circuit, a harmonic current will flow in it. So in a circuit containing a condenser and subject to a sinusoidal *E.M.F.*, the current flow will be sinusoidal. This flow will be in the positive direction

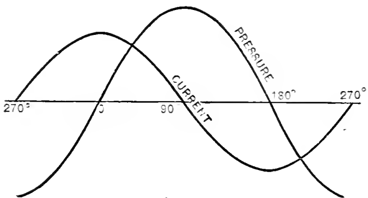


Fig. 25.

from the negative maximum to the positive maximum of pressure, and in a negative direction from the positive maximum to the negative maximum, as described above. This necessitates that the zero values of current occur at the maximum values of pressure; and since the curves are both sinusoids, their relation may be plotted as in Fig. 25. It is immediately seen that these curves are at right angles, as described in § 5, and that the current leads the pressure by 90°.

Reference again to the hydraulic analogy will show that the condenser is completely charged at the instant of maximum positive pressure, discharged at the instant of zero pressure, charged in the opposite direction at the in-

stant of maximum negative pressure, and finally discharged at the instant of the next zero pressure. Thus the charge is zero at the maximum current flow, and at a maximum at zero current, that is, when the current turns and starts to flow out. These points are marked in Fig. 26.

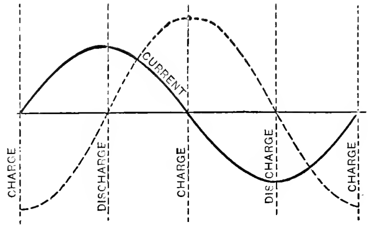


Fig. 26.

21. Current and Voltage Relations. — If a sinusoidal pressure E of frequency f be impressed upon a condenser, the latter is charged in $\frac{1}{4} \cdot \frac{1}{f}$ seconds, discharged in the next $\frac{1}{4f}$ seconds, and charged and discharged in the opposite direction in the equal succeeding intervals. The maximum voltage $E_m = \sqrt{2}E$ (§ 4), hence the quantity at full charge is

$$Q_m = \sqrt{2}\, EC.$$

The quantity flowing through the circuit per second is

$$4fQ_m = 4f\sqrt{2}\, EC.$$

This number therefore represents the average current, or

$$I_{av} = 4\sqrt{2} fEC.$$

From § 4, the effective current

$$I = \frac{\pi}{2\sqrt{2}} I_{av},$$

whence

$$I = 2\pi fCE,$$

and

$$E = \frac{1}{2\pi fC} I.$$

The last is an expression for the volts necessary to send the capacity current through a circuit. The expression $\frac{1}{2\pi f C}$ is called the *capacity reactance* of the circuit. It is analogous to $2\pi f L$, the inductance reactance of an inductive circuit.

If the circuit contain both a resistance R and a capacity C, the voltage E impressed upon it must be considered as made up of two parts, E_r, which sends current through the resistance and is therefore in phase with the current, and E_c, which balances the counter pressure of the condenser and is therefore 90° behind the current in phase.

By Ohm's Law

$$E_r = RI,$$

and from above

$$E_c = \frac{1}{2\pi f C} I.$$

The impressed E must overcome the resultant of these two $E.M.F.$'s; and since they are at right angles

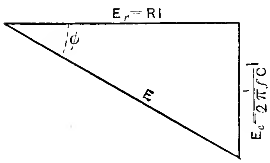


Fig. 27.

$$E = \sqrt{E_r^2 + E_c^2},$$

or

$$I = \frac{E}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}}.$$

The relation of the $E.M.F.$'s is shown graphically in Fig. 27, where the current, which is in phase with the pressure E_r, is seen to lead the impressed pressure by the angle ϕ.

22. Resistance, Inductance, and Capacity in an Alternating-Current Circuit. — The general case of an alter-

nating-current circuit is one that contains resistance, inductance, and capacity. To derive the expression for current flow in such a circuit, it is but necessary to combine the results already found; and this is most readily done graphically. In § 14 it was shown that the counter

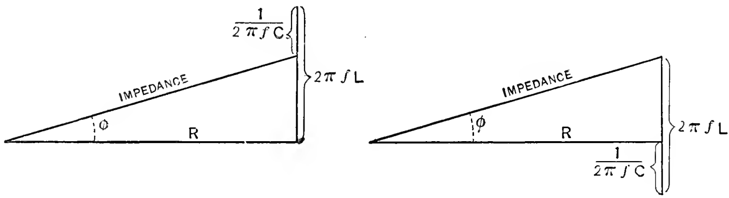


Fig. 28.

E.M.F. due to the inductance reactance of a circuit is $2\pi fL$, and leads the current by 90°. In §21 it was shown that the *E.M.F.* of capacity reactance of a circuit is $\frac{1}{2\pi fC}$ and lags behind the current by 90°. These two *E.M.F.*'s are, then, in exactly opposite phases, or 180° apart, and

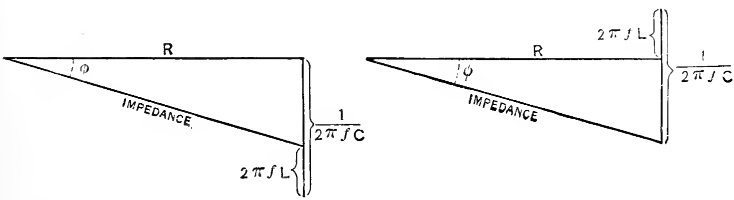


Fig. 29.

the resultant reactance is merely their numerical difference. These relations are shown in Fig. 28, where the reactance of inductance is greater than that of condensance, and in Fig. 29, where the latter exceeds the former, the resistance being the same in either case. Clearly the impedance resulting from the three factors *R*, *L*, and *C* is represented

in direction and in magnitude by the hypothenuse as shown, and the impressed pressure is proportional to this quantity.

The general expression for the flow of an alternating current through any kind of circuit is therefore

$$I = \frac{E}{\sqrt{R^2 + \left[2\,\pi f L - \frac{1}{2\,\pi f C}\right]^2}},$$

the quantity within the brackets indicating an angle of lag of current if positive, and an angle of lead if negative.

23. **Resonance.** — If in a circuit containing inductance and capacity as well as resistance, the two former are proportioned so that

$$2\,\pi f L = \frac{1}{2\,\pi f C},$$

the expression

$$I = \frac{E}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}},$$

reduces to

$$I = \frac{E}{R},$$

the capacity being of a proper magnitude to balance inductance. At one instant energy is being stored in the field at the same rates it is being given to the circuit by the condenser, and at another instant energy is being released from the field at the same rate as it is being stored in the condenser.

When this condition prevails, *resonance* is said to be attained, or the circuit is said to be *in tune*.

If the capacity and inductance be in parallel, enormous

currents may flow between the two. This is because the two are balanced, and the one is at any time ready to receive the energy given up by the other; and a surging once started between them receives periodical increments of energy from the line. This is analogous to the well-known mechanical phenomena that a number of gentle, but well-timed, mechanical impulses can set a very heavy suspended body into violent motion. The frequency of these impulses must correspond exactly to the natural period of oscillation of the body.

If the capacity and inductance be in series, the difference of potential between the terminals of either may be far greater than the *E.M.F.* impressed upon the circuit.

In the first case damage is likely to result from the overloading of the conductors between the inductance and the capacity, even to burning them out, while in the second case the pressure may rise to such a point as to puncture the insulation of all the apparatus in the circuit, even that of the generator itself.

CHAPTER IV.

PROBLEMS ON ALTERNATING-CURRENT CIRCUITS.

24. Definitions of Terms. — In considering the flow of alternating currents through series circuits and through parallel circuits, continual use must be made of various expressions, some of which have been defined during the development of the previous chapters. For convenience the names of all the expressions connected with the general equation

$$I = \frac{E}{\sqrt{R^2 + \left(2\pi f L - \frac{1}{2\pi f C}\right)^2}}$$

will be given and defined.

I is the *current* flowing in the circuit. It is expressed in amperes, and lags behind or leads the pressure, by an angle whose value is

$$\phi = \tan^{-1}\frac{2\pi f L - \frac{1}{2\pi f C}}{R}.$$

E is the harmonic *pressure*, of maximum value $\sqrt{2}\,E$, which is applied to the circuit, and has a frequency f. It is expressed in volts.

R is the *resistance* of the circuit, and is expressed in ohms. It is numerically equal to the product of the impedance by the cosine of ϕ.

L is the *inductance* of the circuit, and is expressed in henrys.

C is the localized *capacity* of the circuit, and is expressed in farads.

$2\pi fL$ is the *inductive reactance* of the circuit, and is expressed in ohms.

$$\frac{1}{2\pi fC}$$

is the *capacity reactance*, or *capacitance*, of the circuit, and is expressed in ohms.

$$\left(2\pi fL - \frac{1}{2\pi fC}\right)$$

is the *reactance* of the circuit, and is expressed in ohms. It is numerically equal to the product of the impedance by the sine of ϕ.

$$\sqrt{R^2 + \left[2\pi fL - \frac{1}{2\pi fC}\right]^2}$$

is the *impedance* or apparent resistance of the circuit, and is expressed in ohms.

$$\frac{1}{\sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}}$$

the reciprocal of the impedance, is the *admittance* of the circuit. It is expressed in terms of a unit that has never

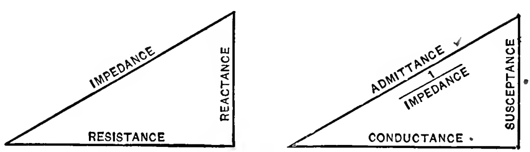


Fig. 30.

been officially named, but which has sometimes been called the mho. There are two components of the admittance, as shown in Fig. 30.

The *conductance* of a circuit is that quantity by which E must be multiplied to give the component of I parallel to E. It is measured in the same units as the admittance, and is numerically equal to

$$\frac{\cos \phi}{\text{impedance}},$$

and also to

$$\frac{R^2}{R^2 + \left(2\,\pi f L - \dfrac{1}{2\,\pi f C}\right)^2}.$$

The *susceptance* of a circuit is that quantity by which E must be multiplied to give the component of I perpendicular to E. It is measured in the same units as the admittance, and is numerically equal to

$$\frac{\sin \phi}{\text{impedance}},$$

and also to

$$\frac{\left(2\,\pi f L - \dfrac{1}{2\,\pi f C}\right)^2}{R^2 + \left(2\,\pi f L - \dfrac{1}{2\,\pi f C}\right)^2}.$$

It should be noticed that while admittance is the reciprocal of impedance, conductance is not the reciprocal of resistance, nor is susceptance the reciprocal of reactance. This becomes evident, upon considering numerical values in connection with the impedance right-angled triangle, *e.g.*, 3, 4, and 5 for the sides.

25. E.M.F.'s in Series. — Alternating *E.M.F.*'s that may be put in series may differ in magnitude, in frequency, in phase relation, and in form or shape of wave. Forms other than that of the sinusoid need not be discussed.

E.M.F.'s of different frequencies in series will give an irregular wave-form whose maximum values will recur at intervals. The duration of these intervals is the least common multiple of the durations of the component half-cycles.

If two harmonic *E.M.F.*'s of the same frequency and phase be in series, the resulting *E.M.F.* is merely the sum of the separate *E.M.F.*'s. This condition is shown in Fig. 31, in which the two *E.M.F.*'s are plotted together, and the resulting *E.M.F.* plotted by making its instantaneous values equal to the sum of the corresponding instantaneous values of the component *E.M.F.*'s. The maximum of the resultant *E.M.F.* is evidently

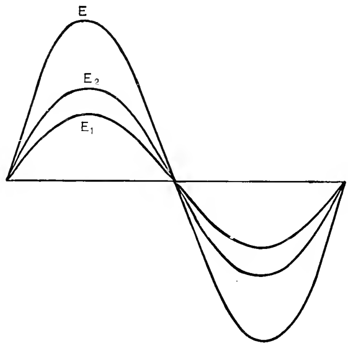


Fig. 31.

$$E_m = E_{1m} + E_{2m},$$

and since $$E = \frac{E_m}{\sqrt{2}}, \quad E_1 = \frac{E_{1m}}{\sqrt{2}},$$

and $$E_2 = \frac{E_{2m}}{\sqrt{2}}, \quad E = E_1 + E_2,$$

as was stated.

If two *E.M.F.*'s of the same frequency, but exactly opposite in phase, be placed in series, it may be similarly shown that the resultant *E.M.F.* is the numerical difference of the component *E.M.F.*'s. This case may occur in the operation of motors.

The most general case that occurs is that of a number of alternating *E.M.F.*'s of the same frequency, but of

different magnitudes and phase displacements. The changes in magnitude and phase and the phase relation of the resulting curve of *E.M.F.* are shown in Fig. 32, where recourse is had once again to the harmonic shadowgraph. But two components, E_1 and E_2, are treated, whose phase displacement is ϕ_1. The radii vectors E_{1m} and E_{2m} are laid off from O with the proper angle ϕ_1 between them, and the shadows traced by their extremities are shown in the dotted curves. The instantaneous value of the resultant *E.M.F.* is the algebraic sum of the corresponding in-

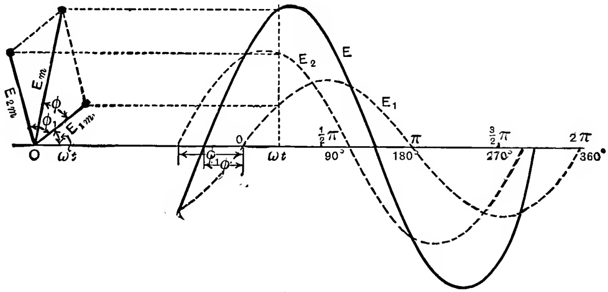


Fig. 32.

stantaneous values of the component *E.M.F.*'s, and the resultant curve of *E.M.F.* is traced in the figure by the solid line. But this solid curve is also the trace of the extremity of the line E_m, which is the vector sum (the resultant of the force polygon) of the component pressures, E_{1m} and E_{2m}. This is evident from the fact that any instantaneous value of the resultant pressure curve is the sum of the corresponding instantaneous values of the component curves, or (§ 3)

$$E' = E_{1m} \sin \omega t + E_{2m} \sin (\omega t + \phi_1).$$

Again from the force polygon

$$E_m \sin (\omega t + \phi) = E_{1m} \sin \omega t + E_{2m} \sin (\omega t + \phi_1).$$

Hence at any instant

$$E' = E_m \sin(\omega t + \phi),$$

wherefore the extremity of the line E_m traces the curve of resultant pressure, ϕ being its angular displacement from E_1. If a third component *E.M.F.* is to be added in series, it may be combined with the resultant of the first two in an exactly similar manner.

So it may be stated as a general proposition, that if any number of harmonic *E.M.F.*'s, of the same frequency, but of various magnitudes and phase displacements, be connected in series, the resulting harmonic *E.M.F.* will be given in magnitude and phase by the vector sum of the component *E.M.F.*'s. The analytic expressions for E and ϕ may be derived by inspection of the diagram, and are

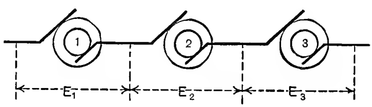


Fig. 33.

$$E = \sqrt{[E_1 \sin\phi_1 + E_2 \sin\phi_2 + \cdots]^2 + [E_1 \cos\phi_1 + E_2 \cos\phi_2 + \cdots]^2},$$

and

$$\phi = \frac{E_1 \sin\phi_1 + E_2 \sin\phi_2 + \cdots}{E_1 \cos\phi_1 + E_2 \cos\phi_2 + \cdots}.$$

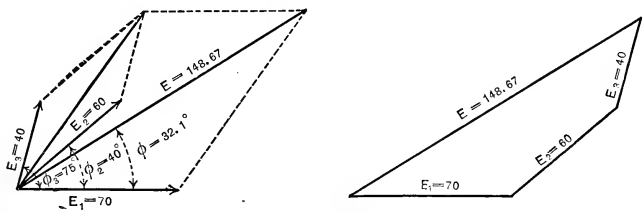


Fig. 34.

As a numerical example, suppose three alternators, Fig. 33, to be connected in series. Suppose these to give sine waves of pressure of values $E_1 = 70$, $E_2 = 60$, and $E_3 = 40$

volts respectively. Considering the phase of E_1 to be the datum phase, let the phase displacements be $\phi_1 = 0°$, $\phi_2 = 40°$, and $\phi_3 = 75°$, respectively. It is required to find E and ϕ. Completing the parallelograms or completing the force polygon as shown in Fig. 34, it is found that $E = 148.7$ volts and $\phi = 32.1°$.

26. Polygon of Impedances. — Consider a circuit having a number of pieces of apparatus in series, each of which may or may not possess resistance, inductance, and capacity. There can be but one current in that circuit when a pressure is applied, and that current must have the same phase throughout the circuit. The pressure at the terminals of the various pieces of apparatus, necessary to maintain through them this current, may, of course, be of different magnitude and in the same or different phases, being dependent upon the values of R, L, and C. Therefore to determine the pressure necessary to send a certain alternating current through such a series circuit, it is but necessary to add vectorially the pressures needed to send such a current through the separate parts of the circuit. This is readily done graphically.

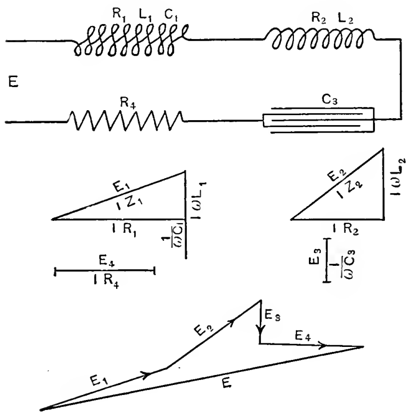


Fig. 35.

Fig. 35 shows the pressures (according to § 22) necessary to send the current I through several pieces of ap-

$E_2 = I Z_2$

$E = I(Z_1 + Z_2 + Z_3 + \cdots$ (vec . . . . .))

paratus, and the combination of these pressures into a polygon giving the resultant pressure E necessary to send the current I through the several pieces in series. In these diagrams, impedance is represented by the letter Z. C_1 and C_3 are localized, not distributed capacities.

For practical purposes, the quantity I, which is common to each side of the triangle, may be omitted; and merely the impedances may be added vectorially in a "polygon of impedances," giving an equivalent impedance, which, when multiplied by I, gives E.

Inspection of the figure shows that the analytical expression for the required E is

$$E = I\sqrt{(R_1 + R_2 + \cdots)^2 + \left[\omega(L_1 + L_2 + \cdots) - \left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2} + \cdots\right)\right]^2}.$$

The pressure at the terminals of any single part of the circuit is

$$E_1 = I\sqrt{R_1^2 + \left[\omega L_1 - \frac{1}{\omega C_1}\right]^2},$$

$$E_2 = I\sqrt{R_2^2 + \left[\omega L_2 - \frac{1}{\omega C_2}\right]^2},$$

$$E_3 = \cdots.$$

It is evident that

$$E_1 + E_1 + \cdots > E,$$

and it is found by experiment that the sum of the potential differences, as measured by a voltmeter, in the various parts of the circuit, is greater than the impressed pressure.

27. A Numerical Example Applying to the Arrangement Shown in Fig. 35. — Suppose the pieces of apparatus to have the following constants:

$R_1 = 85$ ohms,	$L_1 = .25$ henry,	$C_1 = .000018$ farad (18 mf.)
$R_2 = 40$ ohms,	$L_2 = .3$ henry,	. . . .
. . . .	. . . .	$C_3 = .000025$ farad,
$R_4 = 100$ ohms.	. . . .	. . . .

With a frequency of 60 cycles — whence $\omega = 377$ — it is required to find the pressure necessary to be applied to the circuit to send 10 amperes through it.

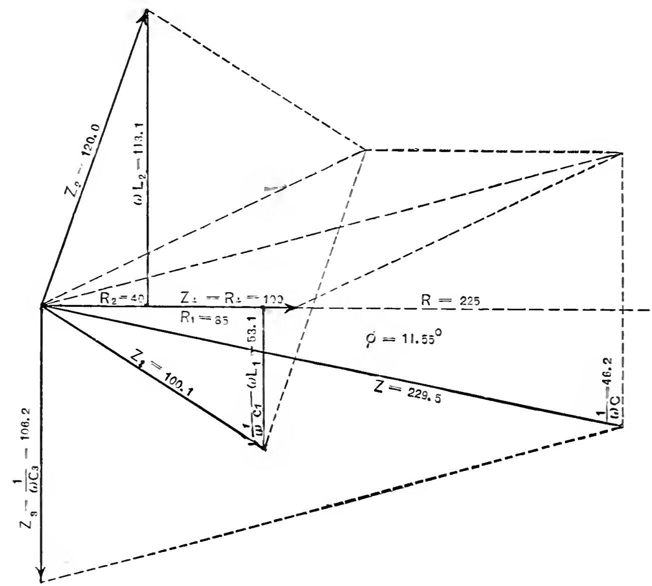


Fig. 36.

The completion of the successive parallelograms in Fig. 36, is equivalent to completing the impedance polygon, and the parts are so marked as to require no explanation. The solution shows that the equivalent impedance, $Z = 229.5$ ohms, that the equivalent resistance (= actual resistance in series), $R = 225$ ohms, that the equivalent reactance is condensive and equals 46.2 ohms, and that $\phi =$

11.55° of lead. Hence the pressure required to send 10 amperes through the circuit is

$$E = 10 \times 229.5 = 2295 \text{ volts.}$$

To obtain the same results analytically

$$E = 10\sqrt{[85+40+100]^2 + [(147.3-94.2)-113.1+106.2]^2},$$

$$E = 2295 \text{ volts.}$$

The voltages at the terminals of the various pieces of apparatus are:

$$\begin{aligned}
E_1 &= 10\sqrt{85^2+(147.3-94.2)^2} &&= 1001 \text{ volts,}\\
E_2 &= 10\sqrt{40^2+113.1^2} &&= 1200 \quad\text{"}\\
E_3 &= 10\sqrt{0^2+106.2^2} &&= 1062 \quad\text{"}\\
E_4 &= 10\sqrt{100^2+0^2} &&= 1000 \quad\text{"}\\
E_1+E_2+E_3+E_4 & &&= 4263 \quad\text{"}
\end{aligned}$$

which is greater than $E = 2195$ volts, showing that the *numerical* sum of the pressures is greater than the impressed pressure; while the vectorial sum of the separate pressures is equal to the impressed pressure.

28. Polygon of Admittances. — If a group of several impedances, Z_1, Z_2, etc., be connected in parallel to a common source of harmonic *E.M.F.* of E volts, their equivalent impedance is most easily determined by considering their admittances Y_1, Y_2, etc. The currents in these circuits would be

$$I_1 = EY_1,$$
$$I_2 = EY_2.$$

The total current, supplied by the source, would be the vector sum of these currents, due consideration being given to their phase relations. Calling this current I, the equation $I = EY$ can be written, where Y is the equivalent admit-

$\therefore I = (I_1 + I_2 + \cdots (\text{vect.})) = E(Y_1 + Y_2 + \cdots (\text{vectorially}$

$= \frac{E}{Z_2} = E Y_2$

tance of the group. To determine Y, a geometrical addition of Y_1, Y_2, etc., must be made, the angular relations being the same as the phase relations of I_1, I_2, etc., respectively. The value of the equivalent admittance may therefore be represented by the closing side of a polygon, whose other sides are represented in magnitude by the several admittances Y_1, Y_2, etc., and whose directions are determined by the phase angles of the currents I_1, I_2, etc., flowing through the admittances respectively. The equivalent impedance then is equal to the reciprocal of Y. The sum of the instantaneous values of the currents in the branch circuits is equal to the corresponding instantaneous values

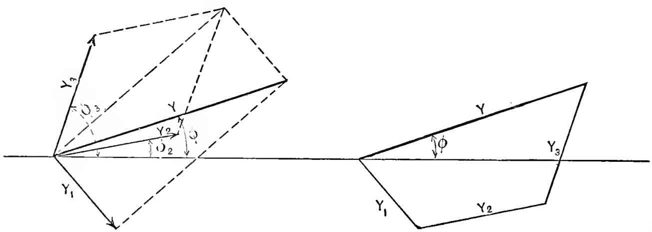


Fig. 37.

in the supply main. As, however, the maximums occur at various times, the sum of the effective currents in the branches is generally greater than the main supply current.

Fig. 37 is a polygon of admittances, showing the method of obtaining the admittance Y and its phase angle referred to a datum line, which is equivalent to a number of parallel admittances, Y_1, Y_2, and Y_3, with angles ϕ_1, ϕ_2, ? and ϕ_3, respectively.

By taking its reciprocal, the equivalent admittance can be transformed into the equivalent impedance. A convenient dimensional scale should be employed. The impedance may be resolved into its equivalent reactance

and its equivalent resistance. The equivalent resistance is not the resistance of the parallel arrangement as measured by direct-current methods.

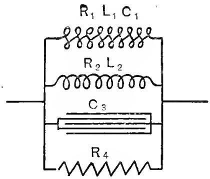


Fig. 38.

As a numerical example, consider the same apparatus as was used in the preceding example, § 27, to be arranged in parallel, as in Fig. 38. All the other conditions and values are as stated before. It is required to find the current that will flow through the mains when ten volts are impressed on the circuit. The diagram, Fig. 39, is self-explanatory. The solution shows that the equivalent admittance $Y = .0224$ and that $\phi = 16.1°$. From this the equivalent impedance

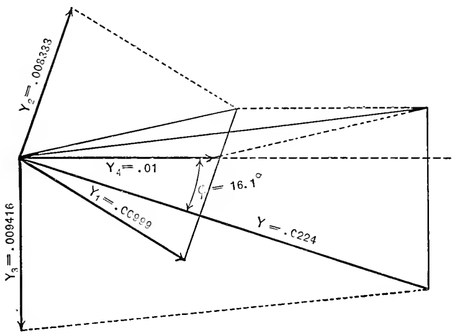


Fig. 39.

$$Z = \frac{1}{.0224} = 44.6 \text{ ohms,} \checkmark$$

the equivalent reactance

$$\left[\omega L - \frac{1}{\omega C}\right] = Z \sin\phi = 12.4 \text{ ohms,} \checkmark$$

and the equivalent resistance $R = Z\cos\phi = 42.9$ ohms. The current that will flow under a pressure of 10 volts is

$$I = EY = 10 \times .0224 = .224 \text{ amperes.}$$

If a circuit have some impedances in series and some in parallel, or in any series parallel combination, the equivalent impedance can always be found by finding the equivalent impedances of the several groups, and then combining these equivalent impedances to get the total equivalent impedance sought.

CHAPTER V.

ALTERNATORS.

29. **Single-phase Alternators.** — As is the case with direct-current machines, alternators have a field and an armature. The direct-current machine's commutator is replaced, in the single-phaser, by a pair of slip-rings; and the current, instead of being rectified, is lead out as alternating current by brushes playing on the rings, as described in § 29, vol. i. Revolving field and inductor alternators differ from this arrangement, as will be shown hereafter.

It is necessary that all but the very smallest alternators should be multipolar to fit them to commercial requirements. For alternators must have in general a frequency between 25 and 125 cycles per second; the armature must be large enough to dissipate the heat generated at full load without its temperature rising high enough to injure the insulation; and finally, the peripheral speed of the armature cannot safely be made to greatly exceed a mile a minute. With these restrictions in mind, and knowing that a point on the armature must pass under two poles for each cycle, it becomes evident that alternators of anything but the smallest capacity must be multipolar.

In practice it is quite as common to have the field of an alternator revolve inside the armature as to have the

Cycles per sec = f.

armature revolve. In a few instances, notably at Niagara, the fields revolve outside the armature. The chief advantage of the revolving field is that it avoids the collection of high-tension currents through brushes, since the armature may be permanently connected up, and only low-tension direct current need be fed through the rings to the field. Other advantages are increased room for armature insulation, and, in polyphasers, the necessity for only two instead of three or more slip-rings.

30. Polyphase Alternators. — Single-phase currents are satisfactory for lights, but not, as yet, for power. As polyphase currents are equally well adapted to both purposes, and since they are generally more economical of transmission than the single-phase, they are much more generally employed. If a motor be operated on a single-phase circuit, the supply of power to it is pulsating. These pulsations occur with great rapidity, there being in the case of unit power factor two for each cycle. A single-phase motor must be larger for the same capacity, than a polyphase motor.

Windings for any number of circuits or *phases* may be placed on a single-armature core, and these may each be separately connected to an outside circuit through slip-rings, or they may be connected together in the armature according to some scheme whereby one slip-ring will be common to two phases. These windings can be placed so that the *E.M.F.*'s generated therein will have any desired phase relations with each other. It is customary to place them so that the *E.M.F.*'s of a two-phase or four-phase system are 90° apart, of a three-phase system are 120° apart, of a six-phase system are 60° apart.

In the following diagrams the curled lines are supposed to represent armature windings, which revolve in a bipolar field. In some cases they are supposed to be wound on cores so as to form pole armatures and in the other cases to form ring armatures. The dots at the terminals represent points of transition between slip-rings and brushes, which are in connection with line wires. It is desirable to consider the relations between the *E.M.F.*'s generated in the armature coils and the pressure between the line-wires, as well as between the currents in the armature coils and the currents in the line-wires. The assumption is made that the different phases are equally loaded, both as to current and as to its phase. The system is then said to be balanced. It is further assumed that the effective *E.M.F.* in each armature coil is E volts, and the effective current I amperes.

31. Two-phase Systems. — In the case of two coils and four wires, the pressure is E volts between the wires attached respectively to each coil. There is no connection between the two coils and their wires.

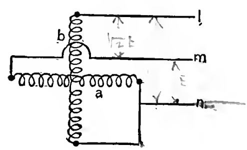


Fig. 40.

In case three wires be employed, as shown in Fig. 40, the pressure between m and n or between l and n is E volts, and $\sqrt{2}E$ volts between l and m. I amperes flows in l and m and $\sqrt{2}I$ in n.

32. Four-phase or Quarter-phase Systems. — When connected, as in Fig. 41*a*, i.e., *star connected*, the pressure between l and m or n and p is $2E$ volts; between n or p and l or m it is $\sqrt{2}E$ volts. The current in each line-wire is I amperes. If connected as in Fig 41*b*, i.e., *ring con-*

nected, the pressure is E volts between l and n, n and m, m and p, or p and l, and $\sqrt{2}E$ volts between l and m, or n and p. The current in each line-wire is $\sqrt{2}I$ amperes.

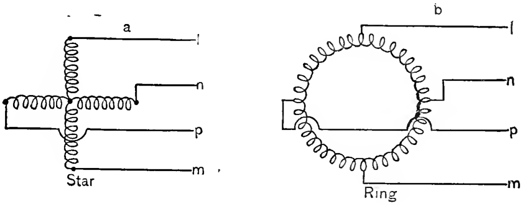
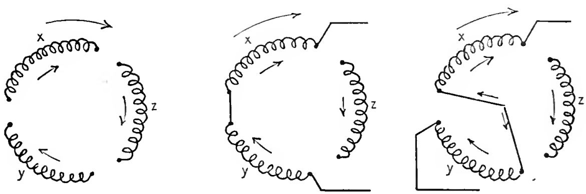


Fig. 41.

33. Three-phase Systems. — The pressure and current relations in three-phase apparatus are often puzzling to the student. Consider three similar coils, x, y, and z, on a ring armature, each covering 120°, as in Fig. 42*a*. The *E.M.F.*'s generated in these coils, when they are rotated in a bipolar field, will have the same maximum values, but



a *b* *c*

Fig. 42.

they will differ in phase from each other by 120°. If two of the coils, x and y, be connected as in *b*, then the pressure between the free terminals would be the result of adding the two *E.M.F.*'s at 120° with each other. If, instead of this connection, the one shown in *c* be made, known as the *star connection* or *Y* connection, the pressure between the free terminals would be the result of

subtracting the *E.M.F.* of coil y from that of x at 120°. Subtraction is necessary because the connections of coil y to the circuit have been reversed. To subtract one quantity from another it is but necessary to change its sign and add. Therefore the pressure between the free terminals is that which results from adding the *E.M.F.*'s of x and y at 300° (= 120° + 180°) as shown in Fig. 43. It is $\sqrt{3}E$ volts. The star connection is generally represented as in Fig. 44, where the pressure between any two line-wires is $\sqrt{3}E$ volts, and the current in each line-wire is I amperes.

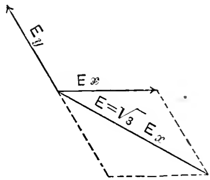


Fig. 43.

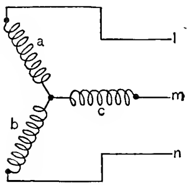


Fig. 44.

If the three coils be connected as in Fig. 45, the result is termed a delta (Δ) or mesh-connection. The pressure between any two of the line-wires is E volts. Each line-wire is supplied with current from two coils, connection being made at the junction between the beginning of one coil and the ending of the other. The value of the current in each wire is $\sqrt{3}I$ amperes. This results from subtracting the current in one coil from that in the other at 120°, which, as before, is the same as adding the currents at 300°.

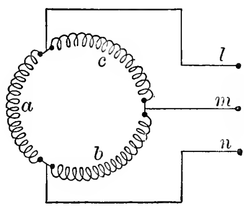


Fig. 45.

The power which is delivered by a three-phase machine is not altered by changing the method of connection. In one case each phase is supplied with I amperes at $\sqrt{3}\,E$ volts, in the other case with $\sqrt{3}\,I$ amperes at E volts.

At any instant the current in one wire of a three-phase system is equal and opposite to the algebraic sum of the currents in the other two wires. This is clearly shown in Fig. 46, where the curve found by adding at each instant the ordinates of two of the three-phase currents is similar, exactly equal, and opposite to the third current.

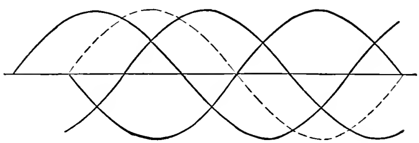


Fig. 46.

34. Electromotive Force Generated. — In § 13, vol. i., it was shown that the pressure generated in an armature is

$$E_{av} = 2\,p\Phi S \frac{V}{60}\,10^{-8},$$

where p = number of pairs of poles,
Φ = maxwells of flux per pole,
V = revolutions per minute,
and S = number of inductors.

In an alternating current $E = k_1 E_{av}$, where k_1 is the form-factor, i.e., the ratio of the effective to the average *E.M.F.* Hence in an alternator yielding a sine wave *E.M.F.*,

$$E = 2.22\,p\Phi S \frac{V}{60}\,10^{-8}.$$

Inasmuch as $p\dfrac{V}{60}$ represents the frequency, f,

$$E = 2.22\,\Phi S f\,10^{-8}.$$

An alternator armature winding may be either concentrated or distributed. If, considering but a single phase, there is but one slot per pole, and all the inductors that are intended to be under one pole are laid in one slot, then

the winding is said to be concentrated, and if the inductors are all in series the above formula for E is applicable. If now the inductors be not all laid in one slot, but be distributed in n more or less closely adjacent slots, the *E.M.F.* generated in the inductors of any one slot will be $\frac{1}{n}$ of that generated in the first case, and the pressures in the different slots will differ slightly in phase from each other, since they come under the center of a given pole at different times. The phase difference between the *E.M.F.* generated in two conductors which are placed in two successive armature slots, depends upon the ratio of the peripheral distance between the centers of the slots to the peripheral distance between two successive north poles considered as 360°. This phase difference angle

$$\phi = \frac{\text{width slot} + \text{width tooth}}{\dfrac{\text{circumference armature}}{\text{no. pairs poles}}} 360.$$

If the inductors of four adjacent slots be in series, and if the angle of phase difference between the pressures generated in the successive ones be ϕ, then letting E_1, E_2, E_3, and E_4 represent the respective pressures, which are

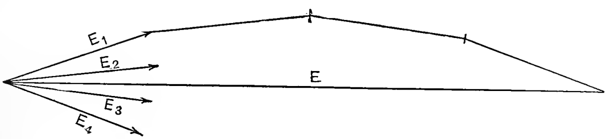


Fig. 47.

supposed to be harmonic, the total pressure, E, generated in them is equal to the closing side of the polygon as shown in Fig. 47. Obviously $E < E_1 + E_2 + E_3 + E_4$. If the winding had been concentrated, with all the induc-

tors in one slot, the total pressure generated would have been equal to the algebraic sum.

The ratio of the vector sum to the algebraic sum of the pressures generated per pole and per phase is called the *distribution constant.* Not only may the number of slots under the pole vary, but they may be spaced so as to occupy the whole surface of the armature between successive pole centers (the peripheral distance between two poles is termed the pole distance), or they may be crowded together so as to occupy only one-half, one-fourth, or any other fraction of this space. Both the number of slots and the fractional part of the pole distance which they occupy affect the value of the distribution constant. A set of curves, Fig. 48, has been drawn, showing the values of this constant for various conditions. Curves are drawn for one slot (concentrated winding), 2, 3, 4 slots in a group, and many slots (i.e., smooth core with wires in close contact on the surface). The ordinates are the distribution constants, and the abscissæ the fractional part of the pole distance occupied by the slots.

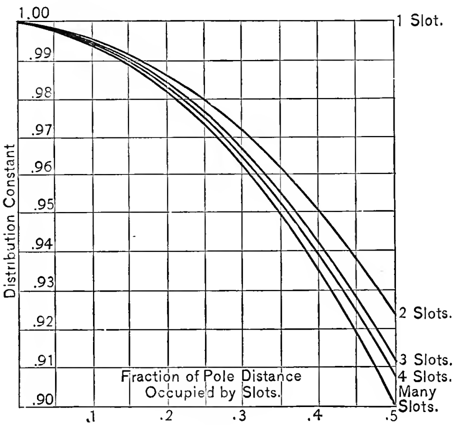


Fig. 48.

The distribution constant, k_2, must be introduced into the formula for the $E.M.F.$ giving

$$E = 2\, k_1 k_2 p \Phi S \frac{V}{60}\, 10^{-8},$$

or, for sine waves,

$$E = 2.22\,k_2\Phi Sf\,10^{-8}.$$

35. Armature Windings. — Some simple diagrams of the windings of multipolar alternators are given in Fig. 49 *et seq.* The first is a single-phase concentrated winding, with the winding which is necessary to make it two-phase in dotted lines. If the two windings be electrically connected where they cross at point P the machine becomes

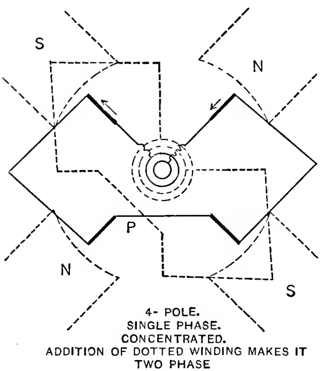


Fig. 49.

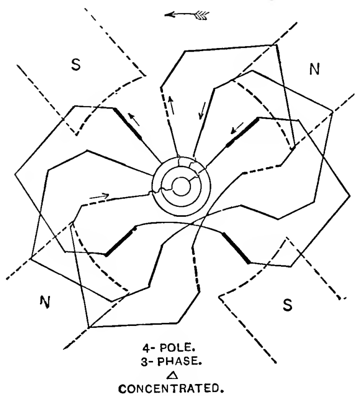
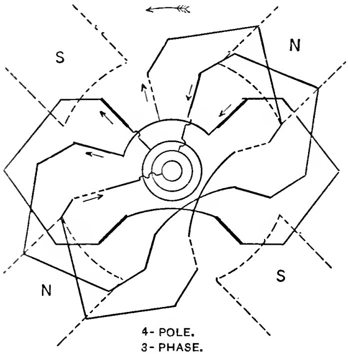


Fig. 50.

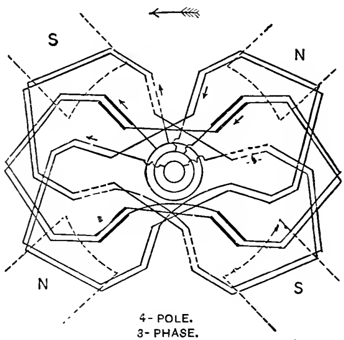
a star-connected four-phaser. Fig. 50 is a three-phase, Δ connected, concentrated winding. Fig. 51 is the same but Y connected. The common junction of the windings would have to be provided with a slip-ring if it were desired to operate a three-phase, four-wire system with the fourth wire connected to the machine. Fig. 52 is a three-phase, Δ connected winding distributed over two slots. In all these diagrams the radial lines represent the inductors; other lines the connecting wires. The inductors of different phases are drawn differently for clearness.

Where but one inductor is shown, in practice there would be a number wound into a coil and placed in the one slot. For simplicity all the inductors of one phase are shown in series. In concentrated windings, all inductors of one



4- POLE.
3- PHASE.
Y.
CONCENTRATED.

Fig. 51.



4- POLE.
3- PHASE.
Δ
DISTRIBUTED.

Fig. 52.

phase carrying current in the same direction could be connected in multiple if desired; but with distributed windings, the coils cannot all be placed in multiple, because the small phase differences between them would set up local currents and give rise to undue heating.

To determine the interior connections for a three-phase Δ winding, place the inductors of a coil of one phase under the centers of the poles, then a maximum pressure in a given direction is generated therein. Since the algebraic sum of the pressures around the Δ must be zero, the other two phases must be connected so that their pressures oppose the first. To determine the Y connection, place the inductors of one phase under the centers of the poles. The *E.M.F.* of this phase will now be at a maximum, say, away from the common center. The other two phases

must be so connected as to have *E.M.F.*'s toward the common center at this instant.

36. Armature Reaction. — The armature reaction of an alternator consists of two parts, distortion and magnetization or demagnetization. These depend upon the armature ampere-turns and upon the lag or lead of the armature current. The maximum pressure is generated in a coil when its opposite inductors are respectively under the centers of north and south poles. This condition is represented in Fig. 53.

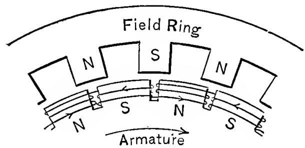


Fig. 53.

If the armature current be in phase with the pressure, I_m in the coils coincides with E_m, and poles on the armature are formed as shown. It is seen that the *M.M.F.*'s both of the field and of the armature conspire to concentrate the flux in the trailing pole tips. So with I in phase with E the armature *M.M.F.* chiefly effects a distortion of the lines, entailing a greater flux density, hence a lower permeability, and also a greater length of air-gap path. This slightly decreases the flux, and affects the regulation of the alternator.

If, now, the current be lagging, the armature will have reached a position in advance, at the instant of maximum current. Therefore, like poles of the field and of the armature will be more directly opposite to each other. The distorting influence will be present in a degree; and there will be considerable demagnetization of the field, due to the opposing *M.M.F.*'s of the armature and field ampere-turns. If the current be leading, then, at the instant of maximum current, a south armature pole will be more

nearly opposite to a north field pole, and their *M.M.F.*'s will be cumulative. The field will be strengthened if the magnetizing reaction exceeds in effect the skewing reaction. Alternators have a much better regulation on non-inductive loads than on inductive loads.

37. Armature Inductance. — The impedance of an alternator armature is made up of its ohmic resistance, R, combined at right angles with its reactance, $2\pi fL$. In practice the inductance, L, is likely to be so great that R becomes negligible, and the impedance equals the reactance. The armature reactance may or may not be an appreciable part of the impedance offered by the completed circuit. If it is appreciable, then the current in the circuit will lag even with a non-inductive load. In any case there will be loss of voltage due to armature impedance which (when R is negligible) is equal to $2\pi fLI$. This is at right angles to the current, and must be properly combined with I times the equivalent impedance of the external circuit to determine the pressure actually generated in the machine. In special cases the armature reactance is the predominant feature of the circuit; for instance, alternators for series arc lighting are made with so great a reactance that the impedance of the external circuit within the limits of operation is negligible in comparison. The alteration in the value of this impedance does not, then, appreciably alter the total impedance of the circuit, and the alternator therefore operates as a *constant-current generator*. Many commercial alternators have sufficient armature reactance to prevent their injuring themselves on dead short circuit for a limited time. It is necessary that armatures should have some considerable inductance

when alternators are to be operated satisfactorily in parallel.

38. Synchronous Reactance. — When an alternator is operating on a load, the pressure, which would be generated on open circuit at the same speed and excitation, is made up of the following parts, and might be found by adding them together in their proper phase relations:

(*a*) terminal voltage, E,

(*b*) ohmic drop in armature, IR, in phase with the current,

(*c*) armature inductance drop, 90° with the current,

(*d*) deficit of actually generated volts due to increase of magnetic reluctance accompanying distortion,

(*e*) deficit or increment of actually generated volts due to the demagnetization of a lagging current or the magnetization of a leading current.

All the parts, except the first mentioned, can be grouped together, and be dealt with collectively by the use of a quantity called the *synchronous impedance*. It is that impedance, which, if connected in series with the outside circuit and an impressed voltage of the same value as the open-circuit voltage at the given speed and excitation, would permit a current of the same value to flow as does flow. This quantity for any load can be determined experimentally with ease. The synchronous impedance has two factors, namely, the armature resistance and a quantity termed the *synchronous reactance*. The two, when combined at right angles, give the synchronous impedance.

Since the synchronous impedance takes account of all the diverse causes of voltage drop above enumerated, it is clear that it has not a physical existence, but is merely a

duction. It is of great use in determining the performance of a machine. Its value is the same for all excitations of the field, but is somewhat different for various loads. These two facts afford a very convenient means of determining its value. Run the alternator at its proper speed. Short-circuit the armature through an ammeter. Excite the field until the ammeter indicates the desired load. Then open the load circuit and read the terminal voltage. The quotient of the volts by the amperes is the synchronous impedance. It may happen that the resistance of the armature is negligibly small, in which case the synchronous reactance equals the synchronous impedance.

30. Saturation Coefficient.— A no-load saturation curve of an alternator may be obtained by measuring the terminal voltage corresponding to various strengths of field current, when the machine is running at its proper speed and without load. Laying off E.M.F.'s, E as ordinates and exciting currents, I_f, as abscissæ, a curve is found as in Fig. 54.

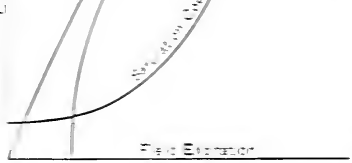


Fig. 54.

The ratio $\dfrac{\dfrac{dI_f}{I_f}}{\dfrac{dE}{E}}$ is called the no-load *saturation coefficient* of the machine.

Another curve, called the load-saturation curve can be obtained by using a variable non-inductive resistance for maintaining the constant full load. The terminal volts corresponding to various field excitations are read on a

voltmeter. This curve will approximately parallel the no-load saturation curve. It will have a zero voltage value for that excitation which causes sufficient voltage to send the full-load current through the synchronous impedance of the armature. A full-load saturation coefficient curve might be drawn from the full-load saturation curve. It will nearly coincide with the other coefficient curve.

These saturation curves have forms similar to magnetization curves for iron. The knee, however, is less abrupt than is general in an iron curve, because of the unvarying permeability of air, and because the different magnetic parts of the generator do not reach saturation at the same time. If the alternator is normally excited to above the knee of the saturation curve, it will require a considerable increase of field current to maintain the terminal voltage when the load is thrown on, while if normally excited below the knee, a slight increase of excitation will suffice. The regulation is, however, better when the magnetization is above the knee; that is, with unaltered field strength, the voltage rise upon throwing off the load is less than if the excitation were below the knee.

40. Leakage Coefficient. — As in direct-current machines, the leakage coefficient of an alternator may be defined as the number of maxwells set up by the field divided by the number of maxwells passing through the armature. It is always greater than unity. Its value depends upon the design of the machine, upon the permeability of the various parts making up the magnetic circuit, upon the load on the machine, and upon the degree of saturation in the fields. In modern commercial machines of size its values lie between 1.1 and 1.5.

41. Efficiency. — The following is abstracted from the Report of the Committee on Standardization of the American Institute of Electrical Engineers. Only those portions are given which bear upon the efficiency of alternators. They will, however, apply equally well to synchronous motors.

The "efficiency" of an apparatus is the ratio of its net power output to its gross power input.

Electric power should be measured at the terminals of the apparatus.

In determining the efficiency of alternating-current apparatus, the electric power should be measured when the current is in phase with the *E.M.F.* unless otherwise specified, except when a definite phase difference is inherent in the apparatus, as in induction motors, etc.

Where a machine has auxiliary apparatus, such as an exciter, the power lost in the auxiliary apparatus should not be charged to the machine, but to the plant consisting of the machine and auxiliary apparatus taken together. The plant efficiency in such cases should be distinguished from the machine efficiency.

The efficiency may be determined by measuring all the losses individually, and adding their sum to the output to derive the input, or subtracting their sum from the input to derive the output. All losses should be measured at, or reduced to, the temperature assumed in continuous operation, or in operation under conditions specified.

In synchronous machines the output or input should be measured with the current in phase with the terminal *E.M.F.* except when otherwise expressly specified.

Owing to the uncertainty necessarily involved in the approximation of load losses, it is preferable, whenever

possible, to determine the efficiency of synchronous machines by input and output tests.

The losses in synchronous machines are:

a. Bearing friction and windage.

b. Molecular magnetic friction and eddy currents in iron, copper, and other metallic parts. These losses should be determined at open circuit of the machine at the rated speed and at the rated voltage, $+ IR$ in a synchronous generator, $- IR$ in a synchronous motor, where $I =$ current in armature, $R =$ armature resistance. It is undesirable to compute these losses from observations made at other speeds or voltages.

These losses may be determined by either driving the machine by a motor, or by running it as a synchronous motor, and adjusting its fields so as to get minimum current input, and measuring the input by wattmeter. The former is the preferable method, and in polyphase machines the latter method is liable to give erroneous results in consequence of unequal distribution of currents in the different circuits caused by inequalities of the impedance of connecting leads, etc.

c. Armature-resistance loss, which may be expressed by $p\ I^2R$; where $R =$ resistance of one armature circuit or branch, $I =$ the current in such armature circuit or branch, and $p =$ the number of armature circuits or branches.

d. Load losses. While these losses cannot well be determined individually, they may be considerable, and, therefore, their joint influence should be determined by observation. This can be done by operating the machine on short circuit and at full-load current, that is, by determining what may be called the "short-circuit core loss."

With the low field intensity and great lag of current existing in this case, the load losses are usually greatly exaggerated.

One-third of the short-circuit core loss may, as an approximation, and in the absence of more accurate information, be assumed as the load loss.

e. Collector-ring friction and contact resistance. These are generally negligible, except in machines of extremely low voltage.

f. Field excitation. In separately excited machines, the I^2R of the field coils proper should be used. In self-exciting machines, however, the loss in the field rheostat should be included.

42. Regulation for Constant Potential. — Alternators feeding light circuits must be closely regulated to give satisfactory service. The pressure can be maintained constant in a circuit by a series boosting transformer, but it is generally considered better to regulate the dynamo by suitable alteration of the field strength.

The simplest method of regulating the potential is to have a hand-operated rheostat in the field circuit of the alternator, when the latter is to be excited from a common source of direct current, or in the field circuit of the exciter, if the alternator is provided with one. The latter method is generally employed in large machines, since the exciter field current is small, while the alternator field current may be of considerable magnitude, and would give a large I^2R loss if passed through a rheostat.

A second method of regulation employs a composite winding, analogous to the compound windings of direct-current generators. This consists of a set of coils; one

on each pole. These are connected in series, and carry a portion of the armature current which has been rectified. The rectifier consists of a commutator, having as many segments as there are field poles. The alternate segments are connected together, forming two groups. The groups are connected respectively with the two ends of a resistance forming part of the armature circuit. Brushes,

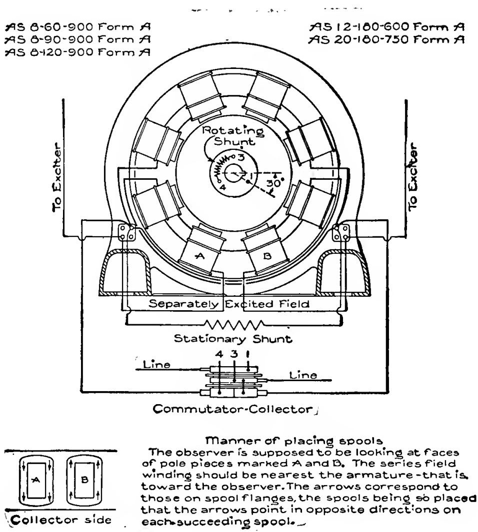


Fig. 55.

bearing upon the commutator, connect with the terminals of the composite winding. The magnetomotive force of the composite winding is used for regulation only, the main excitation being supplied by an ordinary separately excited field winding. The rectified current in the composite coils is a pulsating unidirectional current, that increases the magnetizing force in the fields as the current in the armature increases. The rate of increase is

determined by the resistance of a shunt placed across the brushes. By increasing the resistance of this shunt the amount of compounding can be increased. With such an arrangement an alternator can be over-compounded to compensate for any percentage of potential drop in the distributing lines. The method here outlined is used by the General Electric Company in their single-phase stationary field alternators. The connections are shown in Fig. 55.

A third method of regulation is employed by the Westinghouse Company on their revolving armature alternators, one of which, a 75 k.w. 60~, single-phase machine, is shown in Fig. 56. A composite winding is employed, and the compensating coils are excited by current from a series transformer placed on the spokes of the armature spider. The primary of this transformer consists of but a few turns, and the whole armature current is conducted through it before reaching the collector rings. The secondary of this transformer is suitably connected to a simple commutator on the extreme end of the shaft. Upon this rest the brushes which are attached to the ends of the compensating coil. This commutator is subjected to only moderate currents and low voltages. The current in the secondary of the transformer, and hence that in the compensating coil, is proportional to the main armature current. The machine is wound for the maximum desirable over-compounding, and any less compensation can be secured by slightly shifting the commutator brushes. For there are only as many segments as poles; and if the brushes span the insulation just when the wave of current in the transformer secondary is passing through zero, then the pulsating direct current in the compounding coil

is equal to the effective value of the alternating current; but if the brushes are at some other position, the current will flow in the field coil in one direction for a portion of the half cycle, and in the other direction for the remaining

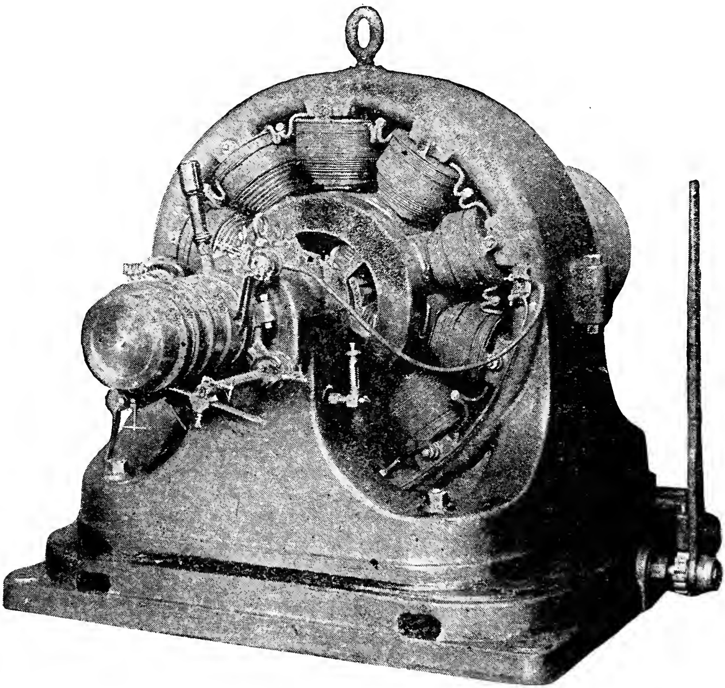


Fig. 56.

portion. A differential action, therefore, ensues, and the effective value of the compensating current is less than it was before.

In order to produce a constant potential on circuits having a variable inductance as well as a variable resist-

ance, the General Electric Co. has designed its compensated revolving field generators, which are constructed for two- or three-phase circuits. The machine, Fig. 57, is of the

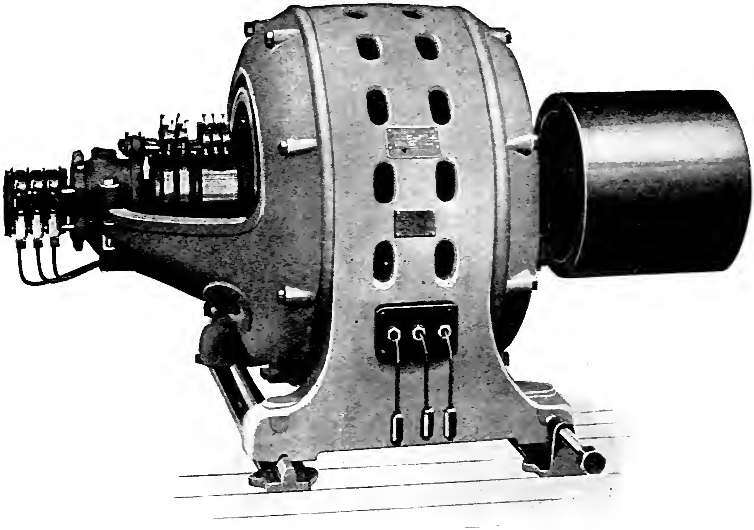


Fig. 57.

revolving field type, the field being wound with but one simple set of coils. On the same shaft as the field, and close beside it, is the armature of the exciter, as shown in Fig. 58. The outer casting contains the alternator armature windings, and close beside them the field of the exciter. This latter has as many poles as has the field of the alternator. Alternator and exciter, therefore, operate in a synchronous relation. The armature of the exciter is fitted with a regular commutator, which delivers direct current both to the exciter field and, through two slip-

rings, to the alternator field. On the end of the shaft, outside of the bearings, is a set of slip-rings, four for a quarter-phaser, three for a three-phaser, through which the exciter armature receives alternating current from one or several series transformers inserted in the mains which lead from the alternator. This alternating current is passed through the exciter armature in such a manner as to cause an armature reaction, as described in § 36, that increases the magnetic flux. This raises the exciter voltage and hence increases the main field current. The

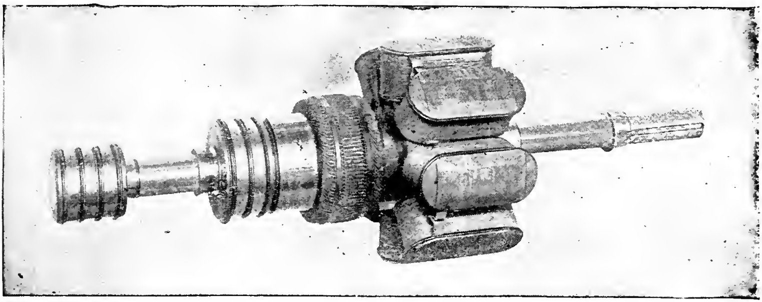


Fig. 58.

reactive *magnetization* produced in the exciter field is proportional to the magnitude and phase of the alternating current in the exciter armature. The reactive *demagnetization* of the alternator field is proportional to the magnitude and phase of the current in the alternator armature. And these currents have the fixed relations of current strength and phase, which are determined by the series transformers. Hence the exciter voltage varies so as to compensate for any drop in the terminal voltage. Neither the commutator nor any of the slip-rings carry pressures of over 75 volts. The amount of over-com-

pounding is determined by the ratio in the series transformers. The normal voltage of the alternator may be regulated by a small rheostat in the field circuit of the exciter.

43. Inductor Alternators. — Generators in which both armature and field coils are stationary are called inductor alternators. Fig. 59 shows the principle of operation of

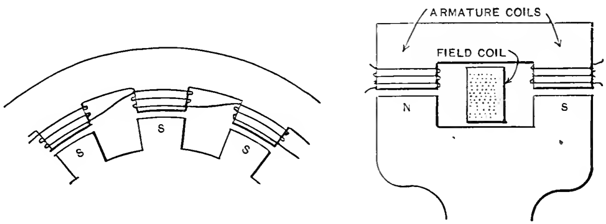


Fig. 59.

these machines. A moving member, carrying no wire, has pairs of soft iron projections, which are called inductors. These projections are magnetized by the current flowing in the annular field coil as shown in figure. The surrounding frame has internal projections corresponding to the inductors in number and size. These latter projections constitute the cores of armature coils. When the faces of the inductors are directly opposite to the faces of the armature poles, the magnetic reluctance is a minimum, and the flux through the armature coil accordingly a maximum. For the opposite reason, when the inductors are in an intermediate position the flux linked with the armature coils is a minimum. As the inductors revolve, the linked flux changes from a maximum to a minimum, but it does not change in sign.

Absence of moving wire and the consequent liability to

chafing of insulation, absence of collecting devices and their attendant brush friction, and increased facilities for

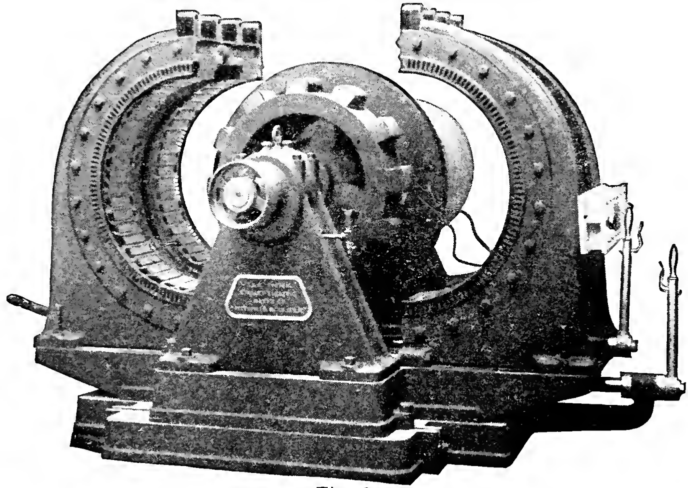


Fig. 60.

insulation are claimed as advantages for this type of machine. By suitably disposing of the coils, inductor alternators may be wound for single- or poly-phase currents.

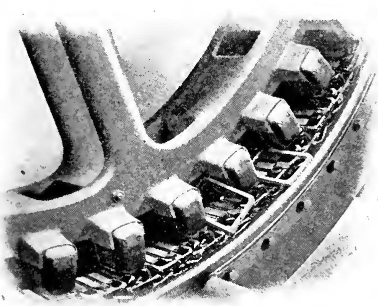


Fig. 61.

The Stanley Electric Manufacturing Company manufacture two-phase inductor alternators. A view of one of their machines is given in Fig. 60, with the frame separated for inspection of the windings. In this picture the field coil is hanging loosely between the pairs

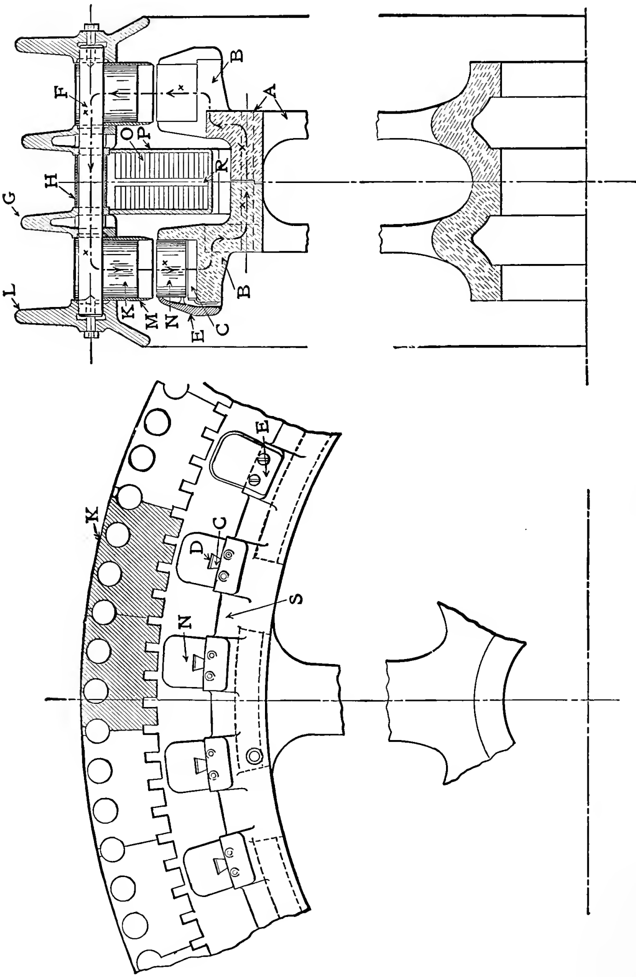


Fig. 62.

of inductors. The theoretical operation of this machine is essentially that described above. All iron parts, both stationary and revolving, that are subjected to pulsations of magnetic flux, are made up of laminated iron. The

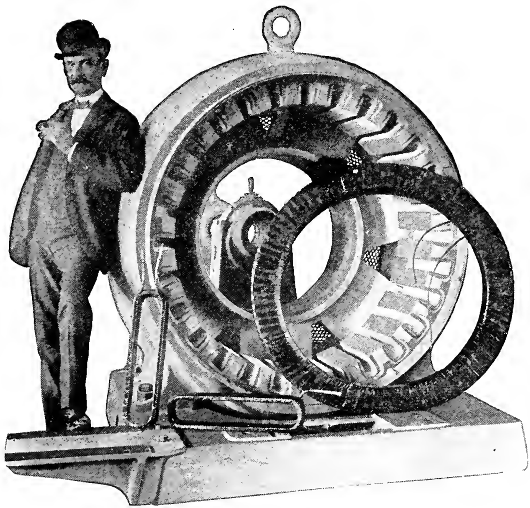


Fig. 63a.

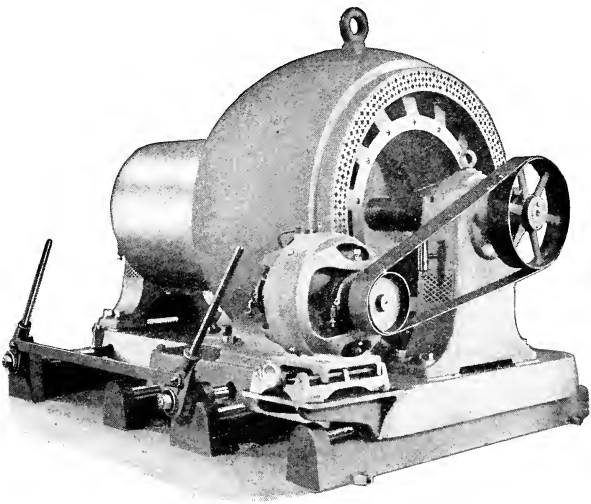


Fig. 63b.

large field coil is wound on a copper spool. Ordinarily when the field circuit of a large generator is broken, the

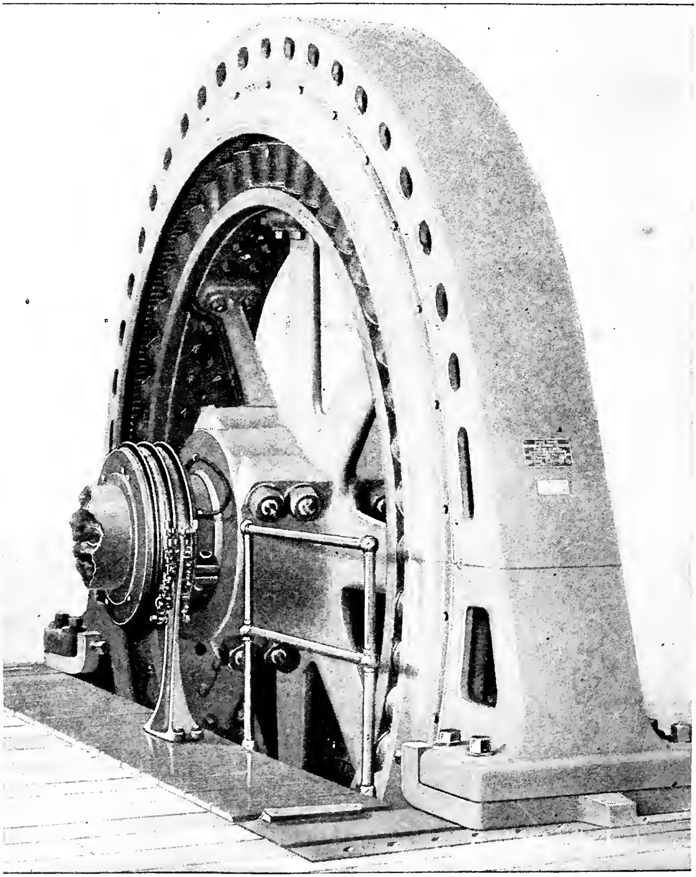


Fig. 64.

E.M.F. of self-induction may rise to so high a value as to pierce the insulation. With this construction the copper spool acts as a short circuit around the decaying flux,

and prevents high *E.M.F.*'s of self-induction. Figs. 61 and 62 show the details of construction of a Stanley machine of a larger size than the one previously shown.

Inductor alternators are also manufactured by Westinghouse and Warren companies. The construction of the machine made by the latter company is shown in Fig. 63*a*. There is but a single field coil, which fits into the recess in the back of the frame as shown. The armature coils surround the pole projections, and the flux through them is altered by the change of reluctance caused by the rotating inductor which carries no wire. The exciter is carried upon a platform which (Fig. 63*b*) forms part of the main frame and is driven by a belt from a pulley on the armature shaft.

44. Revolving Field Alternators. — In this type of alternator, the armature windings are placed on the inside of the surrounding frame, and the field poles project radi-

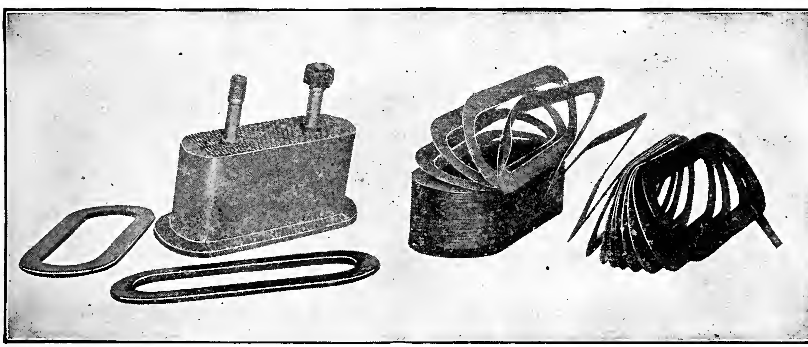
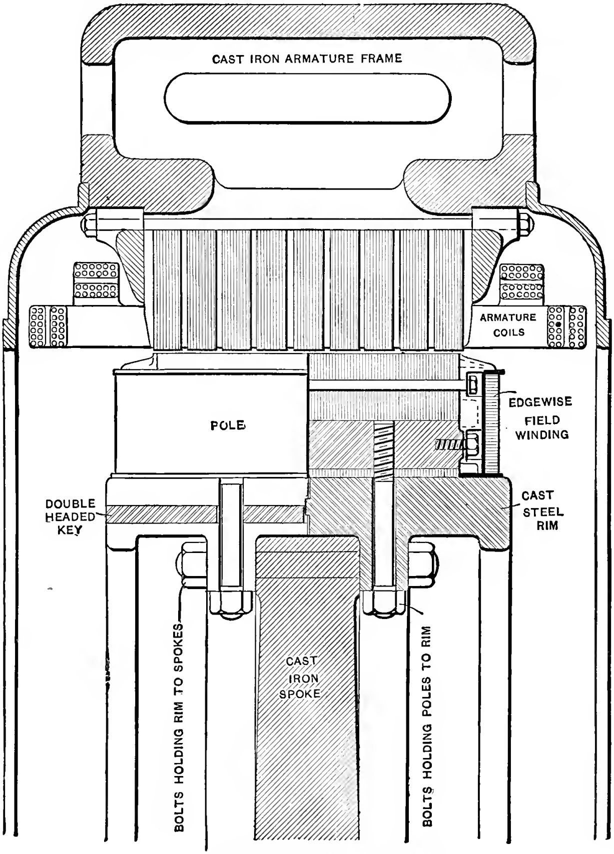


Fig. 65.

ally from the rotating member. As was stated before, this type of construction is to be recommended in the case of large machines which are required to give either high

voltages or large currents. With the same peripheral velocity, there is more space for the armature coils; the



SCALE ¾ INCH EQUALS ONE FOOT.

Fig. 66.

coils can be better ventilated, air being forced through ducts by the rotating field; stationary coils can be more perfectly insulated than moving ones; and the only cur-

rents to be collected by brushes and collector rings are those necessary to excite the fields.

Fig. 64 shows a General Electric 750 K. W. revolving field generator. The two collector rings for the field cur-

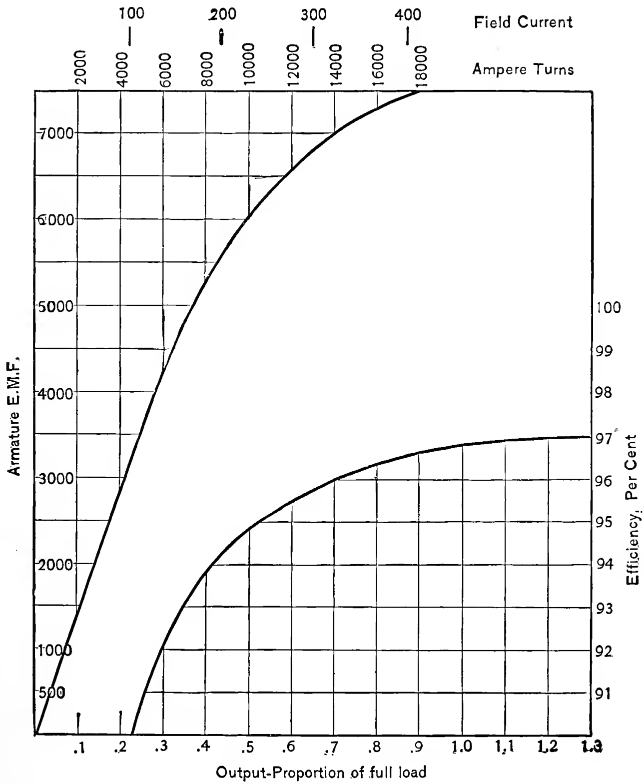


Fig. 67.

rent are shown, and in Fig. 65 the edgewise method of winding the field coils is shown. The collector rings are of cast iron and the brushes are of carbon. Fig. 66 shows the details of construction of a 5,000 K. W. three-phase

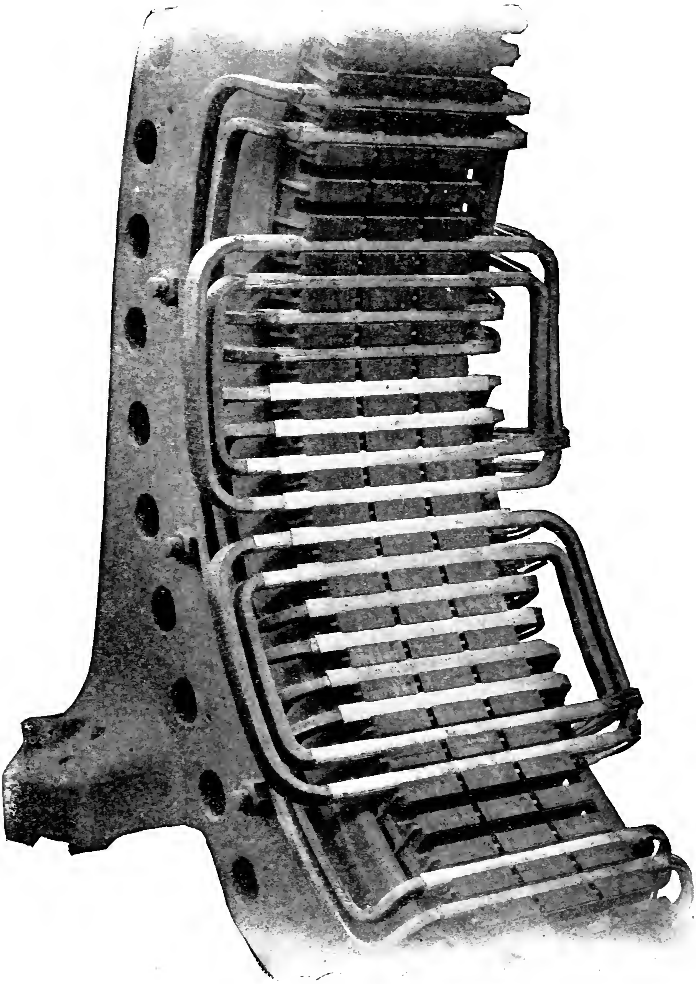


Fig. 68.

6,600-volt machine of this type as constructed for the Metropolitan Street Railway Co. of New York. This machine has 40 poles, runs at 75 R. P. M. at a peripheral

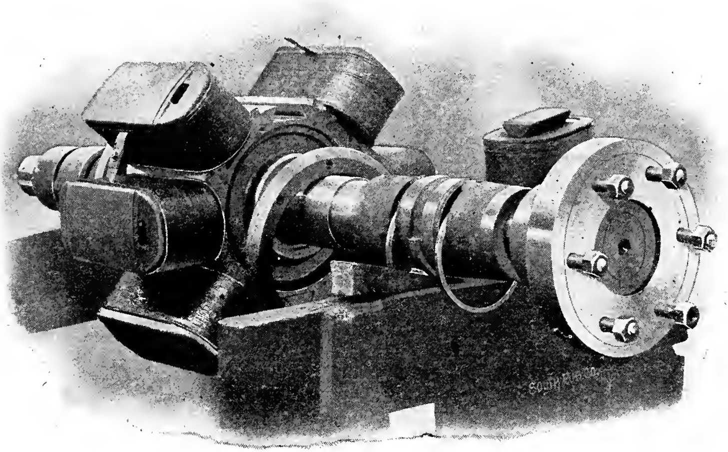


Fig. 69.

velocity of 3,900 feet per minute. This gives a frequency of 25. The air gap varies from five-sixteenths at the pole center to eleven-sixteenths at the tips. The short-circuit current at full-load excitation is less than 800 amperes per leg. The rated full-load current is slightly over 300 amperes. The efficiency and no-load saturation curve is shown in Fig. 67.

The Bullock Electric Mfg. Co. also make generators of this type. The method of placing armature coils is shown in Fig. 68. These coils, as shown, are wire wound, taped, insulated, and held in slots by maple-wood wedges. The field poles are fastened directly to a spider having a heavy rim. The pole pieces are of T-shaped steel punchings,

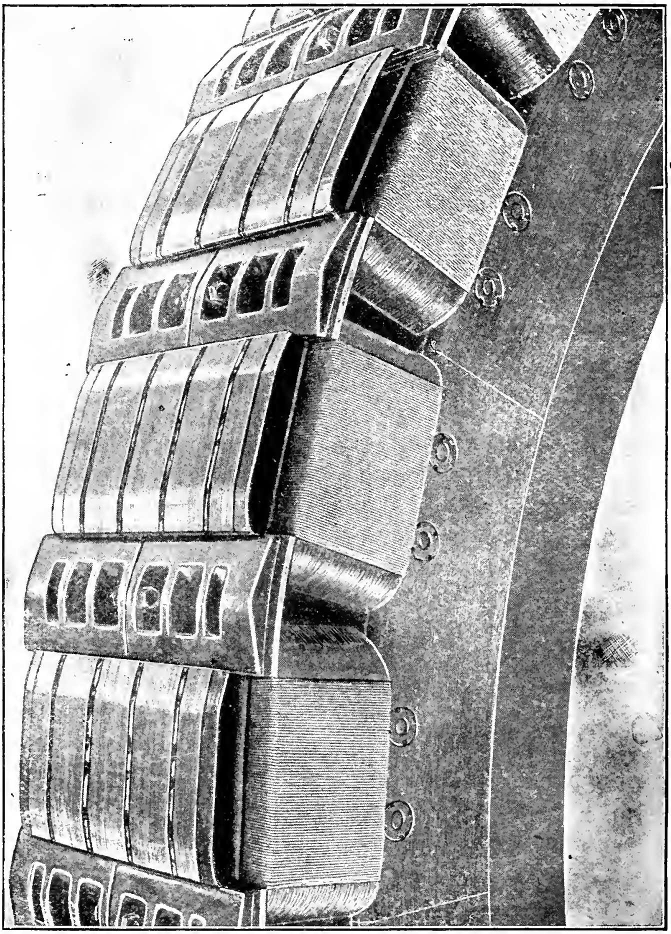


Fig. 70.

held together by rivets and malleable iron end pieces. These are fastened to the rim of the spider by bolts in the case of slow-speed machines, or are dovetailed to fit slots in the rim in the case of high-speed machines. This latter

method of fastening is shown in Fig. 69, which represents the field and shaft of a small-sized high-speed machine.

The Westinghouse rotating field consists of a steel rim mounted upon a cast-iron spider. Into dovetailed slots in the rim are fitted laminated plates with staggered joints. These plates are bolted together. The laminations are supplied at intervals with ventilating ducts. The coils are kept in place by retaining wedges of non-magnetic material. A portion of a field is shown in Fig. 70.

CHAPTER VI.

THE TRANSFORMER.

45. **Definitions.** — The alternating-current transformer consists of one magnetic circuit interlinked with two electric circuits, of which one, the *primary*, receives electrical energy, and the other, the *secondary*, delivers electrical energy. If the electric circuits surround the magnetic circuit, as in Fig. 71, the transformer is said to be of the *core type*. If the reverse is true, as in Fig. 72, the transformer is of the *shell type*. The practical utility of the transformer lies in the fact that, when suitably designed, its primary can take electric energy at one potential, and its secondary deliver the same energy at some other potential; the ratio of the current in the primary to that in the secondary being approximately inversely as the ratio of the pressure on the primary to that on the secondary.

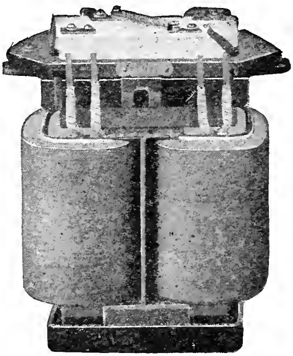


Fig. 71.

The *ratio of transformation* of a transformer is repre-

sented by τ, and is the ratio of the number of turns in the secondary coils to the number of turns in the primary coil. This would also be the ratio of the secondary voltage to

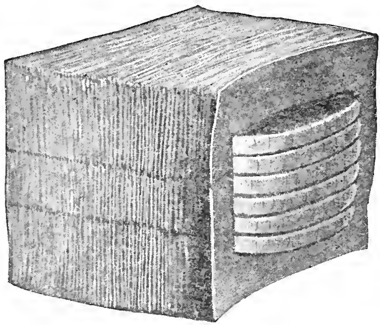


Fig. 72.

the primary voltage, if there were no losses in the transformer. A transformer in which this ratio is greater than unity is called a "step-up" transformer, since it delivers electrical energy at a higher pressure than that at which it is received. When the ratio is less than unity it is called a "step-down" transformer. Step-up transformers find their chief use in generating plants, where because of the practical limitations of alternators, the alternating current generated is not of as high a potential as is demanded for economical transmission. Step-down transformers find their greatest use at or near the points of consumption of energy, where the pressure is reduced to a degree suitable for the service it must perform. The conventional representation of a transformer is given in Fig. 73. In general, little or no effort is made to indicate the ratio of transformation by the relative number of angles or loops shown,

though the low-tension side is sometimes distinguished from the high-tension side by this means.

When using the same or part of the same electric circuit for both primary and secondary, the device is called an auto-transformer. These are sometimes used in the

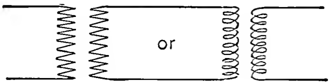


Fig. 73.

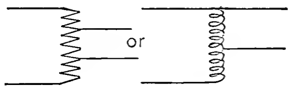


Fig. 74.

starting devices for induction motors, and sometimes connected in series in an alternating-current circuit, and arranged to vary the *E.M.F.* in that circuit. Fig. 74 is the conventional representation of an auto-transformer.

46. Core Flux — (*a*) *Open-circuited secondary.* When the secondary coil of a transformer is open-circuited it is perfectly idle, having no influence on the rest of the apparatus, and the primary becomes then merely a choke coil. A transformer is so designed that its reactance is very high, and its resistance comparatively low. This makes a large impedance, which is almost wholly reactive; hence the current that will flow in the primary when the secondary is open-circuited is very small, and lags practically 90° behind the *E.M.F.* which sends it. This current is called the *exciting current*, or sometimes less properly the magnetizing current or leakage current. A flux is set up in the iron of a transformer, which is sinusoidal and is in phase with the exciting current. This flux induces a practically sinusoidal *E.M.F.* in the primary coil, 90° behind it in phase; because the induced *E.M.F.* is greatest when the time rate of flux change is greatest, and the flux changes fastest as it is passing through the

zero value. This induced *E.M.F.* is 90° behind the flux, which in turn is 90° behind the impressed pressure; therefore the induced *E.M.F.* is 180° behind the impressed *E.M.F.* or is a *counter E.M.F.* The counter pressure is less than the impressed pressure only by the small amount necessary to cause the small exciting current to flow. Neglecting the primary resistance, R_p, and the reluctance, $\mathcal{R}$, of the core, the counter pressure would be equal to the impressed pressure; and in commercial transformers this is true to within a small percentage. Considering that the flux varies sinusoidally, and that its maximum value is Φ_m; then the flux at any time, t, is $\Phi_m \cos \omega t$, and the counter *E.M.F.*, which is equal and opposed to the impressed primary pressure E_p, may be written (§ 13, vol. i.)

$$E_p' = \frac{n_p}{10^8} \frac{d(\Phi_m \cos \omega t)}{dt};$$

and since Φ_m and ω are constant

$$E_p' = 10^{-8} n_p \omega \Phi_m \sin \omega t,$$

from which

$$E_{pm} = 10^{-8} n_p \omega \Phi_m,$$

and

$$\Phi_m = \frac{10^8 E_{pm}}{n_p \omega} = \frac{10^8 \sqrt{2} E_p}{n\ \omega}.$$

This equation is used in designing transformers and choke coils. The values of Φ_m for 60 cycle transformers of different capacities, as determined by experiment and use, are shown in the curve, Fig. 75. It is usual in such designs to also assume a maximum flux density, $\mathcal{B}_m$. While the value assumed differs much with different manufacturers, it is safe to say that for 25 cycles $\mathcal{B}_m$ varies between 9 and 14 kilogausses; for 60 cycles between 6 and 9 kilogausses; and for 125 cycles between 5 and 7

kilogausses. The necessary cross-section, A, of iron, necessary to give the desired counter $E.M.F.$, as well as the number of turns of wire in the primary, is then found from the above, as

$$\Phi_m = \mathfrak{B}_m A.$$

(*b*) *With secondary closed through an outside impedance.* The flux, which is linked with the primary, is also linked with the secondary. Its variations produce in the secon-

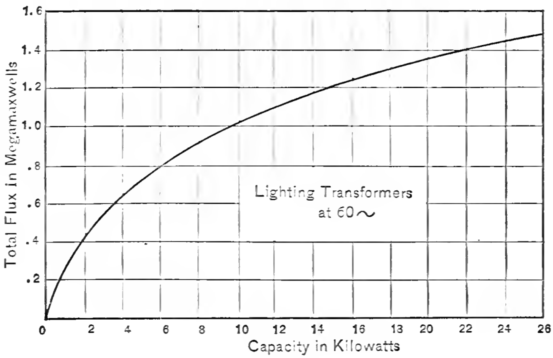


Fig. 75.

dary an $E.M.F.$ τ times as great as the counter $E.M.F.$ in the primary, since there are τ times as many turns in the secondary coil as in the primary, or

$$E_s = \tau E_p.$$

If this secondary be closed through an external impedance, a current I_s will flow through this circuit. In the secondary coil the ampere turns, $n_s I_s$, will be opposed to the ampere turns of the primary, and will thus tend to demagnetize the core. This tendency is opposed by a readjustment of the conditions in the primary circuit. Any demagnetization tends to lessen the counter $E.M.F.$ in the primary coil, which immediately allows more current to

flow in the primary, and thus restores the magnetization to a value but slightly less than the value on open-circuited secondary. Thus the core flux remains practically constant whether the secondary be loaded or not, the ampere turns of the secondary being opposed by a but slightly greater number of ampere turns in the primary. So

$$n_s I_s = n_p I_p, \qquad \text{very nearly,}$$

and

$$I_s = \frac{n_p}{n_s} I_p = \frac{1}{\tau} I_p.$$

The counter *E.M.F.* in the primary of a transformer accommodates itself to variations of load on the secondary in a manner similar to the variation of the counter *E.M.F.* of a shunt wound motor under varying mechanical loads.

If the secondary load be inductive or condensive, then I_s will lag or lead E_s by the same angle that I_p lags or leads E_p, still neglecting R_p, R_s, $\mathcal{R}$, and hysteresis. In such case I_p is 180° from, or opposite to, I_s, and E_p is opposite to E_s. For a more exact statement than the above, see § 54.

47. Equivalent Resistance and Reactance of a Transformer. — If a current of definite magnitude and lag be taken from the secondary of a transformer, a current of the same lag and τ times that magnitude will flow in the primary, neglecting resistance, reluctance, and hysteresis. An impedance which, placed across the primary mains, would allow an exactly similar current to flow as this primary current, is called an *equivalent impedance*, and its components are called *equivalent resistance* and *equivalent reactance*.

If the whole secondary circuit of a transformer with its load have a resistance R_s and a reactance X_s, and if the

primary pressure be E_p and the secondary total pressure E_s, then the current that will flow in the secondary circuit is

$$I_s = \frac{E_s}{\sqrt{R_s^2 + X_s^2}},$$

and it lags behind E_s by an angle ϕ, whose tangent is $\frac{X_s}{R_s}$.

Therefore $$\sqrt{R_s^2 + X_s^2} = \frac{E_s}{{}_s}.$$

If the equivalent impedance have a resistance R and a reactance X then the ratios $\frac{X}{R}$ and $\frac{X_s}{R_s}$ must be equal, since the angle of current lag is the same in both primary and secondary. And since the current in the equivalent impedance has the same magnitude as that in the primary

$$I_p = \frac{E_p}{\sqrt{R^2 + X^2}},$$

and $$\sqrt{R^2 + X^2} = \frac{E_p}{I_p}.$$

But $$I_p = \tau I_s.$$

and $$E_p = \frac{1}{\tau} E_s,$$

therefore, $$\sqrt{R^2 + X^2} = \frac{\frac{E_s}{\tau}}{\tau I_s} = \frac{1}{\tau^2}\frac{E_s}{I_s} = \frac{1}{\tau^2}\sqrt{R_s^2 + X_s^2}.$$

But $$\frac{R}{X} = \frac{R_s}{X_s}.$$

Solving $$R = \frac{1}{\tau^2} R_s,$$

$$X = \frac{1}{\tau^2} X_s,$$

which are the values of the equivalent resistance and reactance respectively.

48. Transformer Losses. — The transformer as thus far discussed would have 100% efficiency, no power whatever being consumed in the apparatus. The efficiencies of loaded commercial transformers are very high, being generally above 95% and frequently above 98%. The losses in the apparatus are due to (*a*) the resistance of the electric circuits, (*b*) reluctance of the magnetic circuit, (*c*) hysteresis, and (*d*) eddy currents. These losses may be divided into core losses and copper losses, according as to whether they occur in the iron or the wire of the transformer.

49. Core Losses. — (*a*) *Eddy current loss.* If the core of a transformer were made of solid iron, strong eddy currents would be induced in it. These currents would not only cause excessive heating of the core, but would tend to demagnetize it, and would require excessive currents to flow in the primary winding in order to set up sufficient counter *E.M.F.*

To a great extent these troubles are prevented by making the core of laminated iron, the laminæ being transverse to the direction of flow of the eddy currents but longitudinal with the magnetic flux. Each lamina is more or less thoroughly insulated from its neighbors by the natural oxide on the surface or by Japan lacquer. The eddy current loss is practically independent of the load.

The *E.M.F.* producing these eddy currents is in phase with the counter *E.M.F.* of the primary coil, both being produced by the same flux. Its value E_e is expressed by the fraction $\frac{P_e}{I_1}$, where P_e is the power loss in watts due to eddy currents, and I_1 is the exciting or no-load primary current. The value of P_e is calculated from the following

empirical formula, in which perfect insulation between the laminæ is assumed:

$$P_e = kvf^2 l^2 \mathfrak{B}_m^2,$$

where

k = a constant depending upon the reluctivity and resistivity of the iron.

v = volume of iron in cm.^{3},

l = thickness of one lamina in cm.,

f = frequency,

and

$\mathfrak{B}_m$ = maximum flux density (Φ_m per cm.^{2}).

In practice k has a value of about 1.6×10^{-11}.

(*b*) *Hysteresis loss.* A certain amount of power, P_h, due to the presence of hysteresis, is required to carry the iron through its cyclic changes. The value of P_h can be calculated from the formula expressing Steinmetz's Law,

$$P_h = 10^{-7} vf\eta \mathfrak{B}_m^{1.6},$$

where

v = volume of iron in cm.^{3},

f = frequency,

$\mathfrak{B}_m$ = the maximum flux density,

and η = the hysteretic constant (.002 to .003).

The portion of the impressed *E.M.F.* which must be expended in the primary circuit to balance the hysteretic loss is

$$E_h = \frac{P_h}{I_1}.$$

This is in phase with I_1.

Closely associated with E_h is another portion of the impressed *E.M.F.* which is consumed in producing the cyclical and sinusoidal variations of magnetic flux. This is not easily considered distinct from E_h. Consider, however, the primary current. There is but one primary current.

At any instant of time a portion of it is balanced and its magnetic effect is neutralized by the demagnetizing current in the secondary; another portion is balanced by the demagnetizing action of the eddy currents; and the remnant is useful in producing the cyclical variations of the magnetic flux. If the flux be sinusoidal this portion of the current cannot be sinusoidal. This is due to the change in permeability with saturation of the iron core. Neither is the rising current curve the reverse of the falling current curve. This is due to the fact that, owing to hysteresis, the permeability on rising flux is smaller than on falling under a given magnetomotive force. This last portion of the primary current is therefore not sinusoidal. As it is but a small percentage of the total current, it is, however, for convenience generally considered as sinusoidal. To send this distorted portion of the primary current requires a portion of the impressed *E.M.F.*, and this is made up of two components, — E_h in phase with the primary current and discussed above, and E_{mag} at right angles with the primary current. This E_{mag} may be considered as sending that portion of the current sufficient to overcome the magnetic reluctance of the core. Being at right angles with I_p it represents no loss of power. During half of the time I_p and E_{mag} have the same direction and during the other half they are in opposite directions. The core therefore alternately receives energy from the circuit and gives it back to the circuit.

To determine the value of E_{mag} consider that it must be of such a magnitude as will send through the primary coil, of resistance R_p, that portion, I_{mag}, of the main current which produces the flux,

$$E_{mag} = R_p I_{mag}.$$

Representing the reluctance of the core by $\mathcal{R}$, and the magnetomotive force necessary to produce the flux Φ_m by $\mathcal{H}$, from §§ 21 and 25, vol. i.,

$$\Phi = \frac{\mathcal{H}}{\mathcal{R}} = \frac{4\,\pi n_p \frac{I_{mag}}{10}}{\mathcal{R}},$$

whence
$$I_{mag} = \frac{10\,\mathcal{R}\Phi}{4\,\pi n_p} = \frac{10\,\mathcal{R}\Phi_m}{4\sqrt{2}\,\pi n_p},$$

and
$$E_{mag} = \frac{10\,R_p\mathcal{R}\Phi_m}{4\sqrt{2}\,\pi n_p}.$$

I_{mag} is called the magnetizing current of a transformer. The primary counter *E.M.F.*, E, is less than the primary line voltage by the slight pressure necessary to send this current through the primary resistance, thus,

$$E = E_p - I_{mag}R_p.$$

The value of $\mathcal{R}$ is calculated (§ 24, vol. i.) from

$$\mathcal{R} = \frac{l}{A}\rho,$$

where l is the length of magnetic circuit, A its cross-section and ρ the reluctivity of the iron

$$\left(\rho = \frac{1}{\mu} = \frac{1}{\text{permeability}}\right).$$

In modern commercial transformers the core loss at 60 ~ may be about 70% hysteresis and 30% eddy current loss. At 125 ~ it may be about 55% hysteresis and 45% eddy current loss. This might be expected, since it was shown that the first power of f enters into the formula for hysteresis loss, while the second power of f enters into the formula for eddy current loss.

The core loss is also dependent upon the wave-form of the impressed *E.M.F.*, a peaked wave giving a somewhat lower core loss than a flat wave. It is not uncommon to find alternators giving waves so peaked that transformers tested by current from them show from 5% to 10% less core loss than they would if tested by a true sine wave. On the other hand generators sometimes give waves so flat that the core loss will be greater than that obtained by the use of the sine wave.

The magnitude of the core loss depends also upon the temperature of the iron. Both the hysteresis and eddy current losses decrease slightly as the temperature of the iron increases. In commercial transformers, a rise in temperature of 40° C. will decrease the core loss from 5% to 10%. An accurate statement of the core loss thus requires that the conditions of temperature and wave-shape be specified.

The core loss is practically constant at all loads, and is the same whether measured from the high-tension or the low-tension side, the exciting current in either case being the same percentage of the corresponding full-load current. The exciting current varies in magnitude with the design of the transformer. In general it will not exceed 5% of the full-load current, and in standard lighting transformers it may be as low as 1%. In transformers designed with joints in the magnetic circuit the exciting current is largely influenced by the character of the joints, increasing if the joint is poorly constructed. In the measurement of core loss, if the product of the impressed volts by the exciting current is less than twice the measured watts (i.e., if $\cos\phi > .5$ or $\phi < 60°$) there is reason to suspect poorly constructed magnetic joints or higher densities in the iron than good practice allows.

50. Copper Losses. — The copper losses in a transformer are almost solely due to the regular current flowing through the coils. Eddy currents in the conductor are either negligible or considered together with the eddy currents in the core.

When the transformer has its secondary open-circuited the copper loss is merely that due to the exciting current in the primary coil, $I^2_{mag} R_p$. This is very small, much smaller than the core loss, for both I_{mag} and R_p are small quantities. When the transformer is regularly loaded the copper loss in watts may be expressed

$$P_c = I_p^2 R_p + I_s^2 R_s.$$

At full load this loss will considerably exceed the core loss. While the core loss is constant at all loads, the copper loss varies as the square of the load.

51. Efficiency. — Since the efficiency of induction apparatus depends upon the wave-shape of *E.M.F.*, it should be referred to a sine wave of *E.M.F.*, except where expressly specified otherwise. The efficiency should be measured with non-inductive load, and at rated frequency, except where expressly specified otherwise.

The efficiency of a transformer is expressed by the ratio of the net power output to the gross power input or by the ratio of the power output to the power output plus all the losses. The efficiency, ϵ, may then be written,

$$\epsilon = \frac{V_s I_s}{V_s I_s + P_h + P_e + P_c},$$

where V_s is the difference of potential at the secondary terminals.

If the transformer be artificially cooled, as many of the

larger ones are, then to this denominator must be added the power required by the cooling device, as power consumed by the blower in air-blast transformers, and power consumed by the motor-driven pumps in oil or water cooled transformers. Where the same cooling apparatus supplies a number of transformers or is installed to supply future additions, allowance should be made therefor.

Inasmuch as the losses in a transformer are affected by the temperature, the efficiency can be accurately specified only by reference to some definite temperature, such as 25° C.

The *all-day efficiency* of a transformer is the ratio of energy output to the energy input during the twenty-four hours. The usual conditions of practice will be met if the calculation is based on the assumption of five hours full-load and nineteen hours no-load in transformers used for ordinary lighting service. With a given limit to the first cost, the losses should be so adjusted as to give a maximum all-day efficiency. For instance, a transformer supplying a private residence with light will be loaded but a few hours each night. It should have relatively much copper and little iron. This will make the core losses, which continue through the twenty-four hours, small, and the copper losses, which last but a few hours, comparatively large. Too much copper in a transformer, however, results in bad regulation. In the case of a transformer working all the time under load, there should be a greater proportion of iron, thus requiring less copper and giving less copper loss. This is desirable in that a loaded transformer has usually a much greater copper loss than core loss, and a halving of the former is profitably purchased even at the expense of doubling the latter.

52. Regulation. — The definition of the regulation of a transformer as authorized by the American Institute of Electrical Engineers is as follows: "In transformers the regulation is the ratio of the rise of secondary terminal voltage from full-load to no-load (at constant impressed primary terminal voltage) to the secondary full-load voltage." Further conditions are that the frequency be kept constant, that the wave of impressed *E.M.F.* be sinusoidal, and that the load be non-inductive.

Not the whole primary impressed pressure is operative in producing secondary pressure, for $I_p R_p$ volts are lost in overcoming the resistance of the primary coil. Besides this there is a flux linked with the primary that does not link the secondary. This induces a counter pressure in the primary which neutralizes a part of the impressed pressure. Such flux, linking one coil but not the other, is called leakage flux. Furthermore, not all of the *E.M.F.* induced in the secondary is utilizable at the terminals. There is a drop of $I_s R_s$ volts due to the resistance of the secondary coil, and another drop due to a leakage flux which links the secondary but not the primary. All these drops increase with load, and therefore, neglecting core loss effects, at no load $E_s = \tau E_p$, but on load, $E_s < \tau E_p$, and the percentage of the full-load secondary pressure represented by this fall is the regulation.

The leakage flux affects the action of a transformer just the same as would an inductance connected in series with the same transformer, the latter having no leakage flux.

The leakage flux increases with the current; and if, for a current I it be Φ, then the value of the inductance, L, is

$$L = \frac{n\Phi}{10^8 I}.$$

where n is the number of turns in the coil. A method of calculating L, the *equivalent inductance*, is given in the next article.

The resistance of the secondary causes a drop $I_s R_s$. The same effect on the regulation would be caused if the secondary resistance were zero and another resistance whose value is $R_3 = \frac{R_s}{\tau^2}$ were inserted in the primary circuit. The imaginary primary drop, resulting from this insertion, has to be but $\frac{1}{\tau}$ as great as the actual secondary drop to be as great a percentage of the impressed E, and there is τ times as much current to cause it, hence $R_3 = \frac{R_s}{\tau^2}$. The power lost in this imaginary resistance is $I_p^2 R_3$, and this equals the power really lost in the secondary $I_s^2 R_s$, since $I_p = \tau I_s$, and $R_3 = \frac{R_s}{\tau^2}$.

In order to calculate the regulation, consider this equivalent of secondary drop to be accounted for in the primary. Then for a given impressed $E.M.F.$ on the primary, E_p, the terminal voltage on the secondary will be

at no load $$E_s = \tau E_p,$$

at primary load I_p,

$$V_s = \tau \left[E_p - I_p(R_p + R_3)\cos\phi - \omega L I_p \sin\phi\right],$$

where $E_s =$ secondary pressure generated,

$V_s =$ difference of potential at secondary terminals,

$L = L_p + L_3$ as calculated in the next section,

and $\phi =$ angle of lag of I_p behind E_p.

Then from the definition of regulation, when I in the above is made equal to the full-load current,

$$\text{Regulation} = \frac{\tau E_p - V_s}{V_s}.$$

53. Calculation of Equivalent Leakage Inductance.— The arrangement of one of the most usual kinds of core type transformers called the "type H," is shown in Fig. 76. The coarse wire is wound inside the fine wire, and as these are more generally used as step-down transformers the latter will be called the primary.

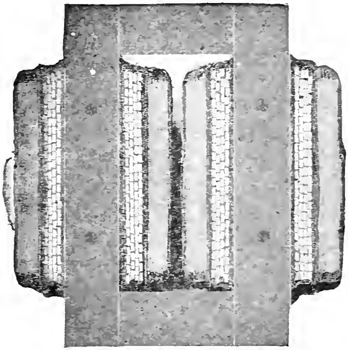


Fig. 76.

Fig. 77 shows one leg of the transformer, giving the paths of leakage flux and the system of notation employed. The discussion is carried on entirely in C. G. S. units. Consider the secondary (coarse wire) coil first.

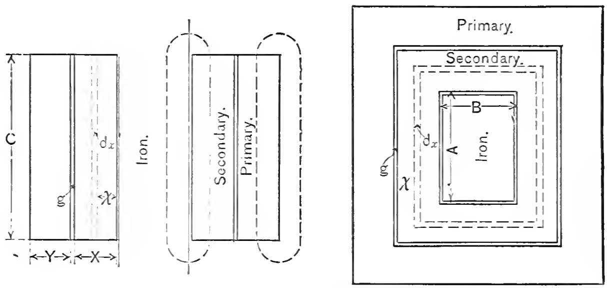


Fig. 77.

The $M.M.F.$ tending to send flux through the elementary portion dx and back through the iron is $\frac{x}{X}$ of the whole $M.M.F.$ of the secondary, so for any element,

$$M.M.F. = 4\pi n_s i_s \frac{x}{X}.$$

Since the permeability of iron is roughly 1000 times that of air, no appreciable error is introduced by considering the whole reluctance of the circuit of the leakage flux to be in the air portion of that circuit. If it be assumed that the lines of force follow a circular path from the end of the coil to the iron, the length of the air portion of the magnetic circuit for any element is $C + \pi x$. The use of this value will result in an integral expression, simple enough in theory, but too unwieldy to be introduced on these pages. Since the portion of the air path outside the coil (the curved portion) is a small part of the whole path, no serious error will be introduced by assuming that the leakage flux from any element follows a path whose length is the average length $C + \pi \frac{X}{2}$. The cross-section area of the air part of the magnetic circuit for any element is $(2A + 2B + 8x)\,dx$. Therefore the reluctance of any element is

$$\mathcal{R} = \frac{C + \frac{\pi}{2} X}{2\,(A + B + 4\,x)\,dx}.$$

The elementary leakage flux, $d\Phi$, is then

$$d\Phi = \frac{M.M.F.}{\mathcal{R}} = \frac{4\,\pi n_s i_s x}{X} \times \frac{2\,(A + B + 4\,x)\,dx}{C + \frac{\pi}{2} X}.$$

Since this flux links with $\frac{x}{X}$ of the secondary turns, the number of linkages is

$$\frac{8\,\pi n_s i_s (A + B + 4\,x)\,x\,dx}{\left(C + \frac{\pi}{2} X\right) X} \times \frac{x n_s}{X} = \frac{8\,\pi n_s^2 i_s (A + B + 4\,x)\,x^2 dx}{\left(C + \frac{\pi}{2} X\right) X^2}.$$

By definition (§ 8) the coefficient of self-induction, l, is numerically equal to the number of linkages per unit current. Therefore

$$dl_s = \frac{\text{linkages}}{i_s} = \frac{8\,\pi n_s^2 (A + B + 4\,x)\,x^2 dx}{\left(C + \frac{\pi}{2} X\right) X^2}.$$

The limits of the variable x are 0 and X, therefore

$$l_s = \frac{8\,\pi n_s^2}{\left(C + \frac{\pi}{2} X\right) X^2} \left[(A + B) \int_0^X x^2 dx + 4 \int_0^X x^3 dx \right].$$

$$l_s = \frac{8\,\pi n_s^2}{\left(C + \frac{\pi}{2} X\right) X^2} \left[\frac{A + B}{3} X^3 + X^4 \right],$$

$$l_s = 8\,\pi n_s^2 X \frac{A + B + 3\,X}{3\left(C + \frac{\pi}{2} X\right)}.$$

This applies to one leg of the transformer. For the two legs, upon reverting to practical units,

$$L_s = \frac{16}{10^9}\,\pi n_s^2 X \frac{A + B + 3\,X}{3\left(C + \frac{\pi}{2} X\right)},$$

all the terms of which are either absolute numbers or linear dimensions in centimeters.

It cannot be objected that this analysis does not take account of the leakage flux that does not travel the whole length of the coil, C. It is a true statement for any length, and therefore might be applied to the elementary length dC, which when integrated would give the result stated above.

The value of L_p is determined in the same way, and the

expression therefore is quite similar. There can be no iron in the path of the leakage flux from the outside coil, so the reluctance will be twice as great. The value that is represented by A for the inner coil becomes $A + 2X + 2g$ for the outer. Likewise B is replaced by $B + 2X + 2g$, g being the space occupied by insulation between the coils. Then

$$L_p = \frac{8}{10^9}\pi n_p^2 Y \frac{(A + 2X + 2g) + (B + 2X + 2g) + 3Y}{3(C + \pi Y)}.$$

If the secondary circuit is open the secondary coil is idle, equivalent to so much air, and all the flux set up by the primary is leakage flux.

As the secondary resistance can be replaced by an equivalent primary resistance, $R_3 = \frac{R_s}{\tau^2}$, for purposes of calculation, so also the secondary inductance can be replaced by an equivalent inductance in the primary, $L_3 = \frac{L_s}{\tau^2}$. These values, L_p and L_3, are to be used in the formula at the end of the last section for determining the regulation of a transformer.

54. Exact Solution of a Transformer. — In the treatment of regulation, efficiency, etc., heretofore, certain small errors have been allowed, due to neglecting the effects of the core, eddy currents, and hysteresis losses. The following graphic solution, adapted from Steinmetz, takes account of all these effects, and is general in all respects.

It must first be understood that there are three fluxes to be considered: (1) The useful flux that links both coils. It is not in any definite phase with either I_p or I_s. It is, however, always at right angles to the *E.M.F.* it induces,

the direct in the secondary, and the counter in the primary. (2) The leakage flux of the primary coil. This links the primary only, and being independent of I_s is always in phase with I_p. (3) The leakage flux of the secondary coil. This is similarly in phase with I_s.

Let $E_s = E.M.F.$ induced in secondary.
V_s = difference of potential at secondary terminals,
E_p = impressed primary pressure.
E_o = operative part of $E_p \left(E_o = \frac{E_s}{\tau}\right)$,
I_p and I_s = primary and secondary currents respectively.
ϕ_p and ϕ_s = lag of primary and of secondary currents respectively behind E_p and E_s.
γ = angle of lag of $M.M.F.$ behind useful flux.

The problem is: Given the necessary data of the transformer, to determine its behavior with any specified load on the secondary.

As in Fig. 78, draw the line Φ, representing the direction of the flux, vertically for convenience. In this analysis, the no-load exciting current is separated into two components. One is used in neutralizing the demagnetizing effect of the eddy currents. The other, I_h, is the magnetizing current and is also made up of two components, one in phase with the primary pressure E_p and the other at right angles with it. The relative magnitudes of these two components are dependent upon the shape of the hysteresis curve of the iron. Once determined they may be represented as $I_h \cos \beta$ and $I_h \sin \beta$ where β is termed the angle of *hysteretic lag*. When multiplied by E_p the first represents the power lost in hysteresis; the second the power passing backward and forward between the

magnetic field and the circuit. If to the former the power lost in eddy currents, W_e, be added and the two be combined with the latter as in Fig. 79 an angle γ results, which

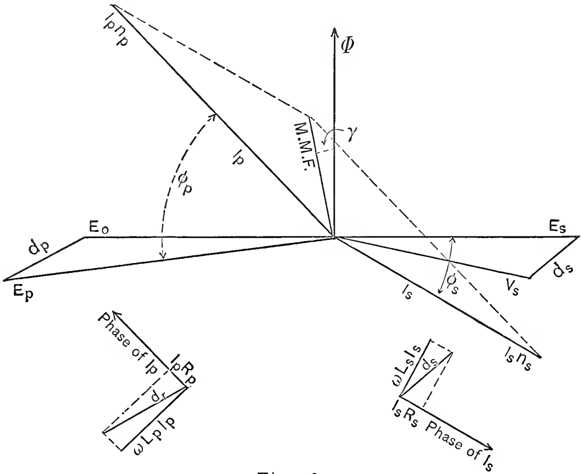


Fig. 78.

represents the lag of the magnetomotive force. Determine the angle γ in this manner. Draw the line *M.M.F.* (Fig. 78) γ ahead of Φ indicating in direction and magnitude the ampere turns which must exist to set up the flux Φ. Its value is determined during the transformer design. Draw from the center the line E_o, 90° ahead of the flux, representing the operative primary pressure. Its length is $\frac{1}{\tau} E_s$ and as it *opposes* the counter primary pressure, it is set ahead of the flux. Draw the line E_s, 90° behind Φ, representing the pressure induced in the secondary. Its length is proportional to the no-load secondary terminal pressure.

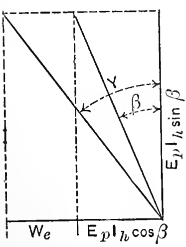


Fig. 79.

The angle ϕ_s, the lag due to the whole secondary circuit, is known. Draw I_s at ϕ_s behind E_s, and extend it till its length is proportional to the secondary ampere turns, $I_s\, n_s$. This line represents one component of the magnetizing force. From this component line and the resultant line $M.M.F.$ determine the other component $I_p\, n_p$. Divide this by n_p and the primary current is discovered in magnitude and phase.

There is a drop of $I_p\, R_p$ volts in the primary. The impressed pressure that compensates for this is in phase with I_p. A counter voltage 90° behind I_p will be set up due to the primary leakage flux. Its value is $\omega L_p\, I_p$. To overcome this an impressed pressure must be supplied opposite it in phase or 90° ahead of the current. In a side figure vectorially add $I_p R_p$ in the phase of I_p and $\omega L_p I_p$ 90° ahead of this phase. This gives the direction and magnitude of the drop d_p in the primary. Properly add d_p to the operative pressure E_o and the necessary impressed pressure E_p is the resultant. The angle between I_p and E_p is the angle of lag ϕ_p of the primary current. It slightly exceeds ϕ_s.

The pressure E_s is generated in the secondary coil. There is a drop of $I_s\, R_s$ volts in this coil in phase with I_s. A counter voltage 90° behind I_s will be set up due to the secondary leakage flux. Its value is $\omega L_s\, I_s$. To overcome this $\omega L_s\, I_s$ volts generated at 180° from this (i.e., 90° ahead of I_s) will be consumed. In a side figure vectorially add $I_s\, R_s$ in the phase of I_s and $\omega L_s I_s$ at 90° ahead of this phase. This gives the drop d_s in the secondary coil. This drop must be subtracted from the pressure generated to give the secondary terminal volts. To subtract a vector, revolve it 180° and proceed as in addition. Properly sub-

tract d_s from E_s and the resultant, V_s, is the potential difference at the terminals of the secondary coil of the transformer.

By constructing this diagram for full load I_s and then for $I_s = 0$, the regulation of a transformer can be found by the ratio of the difference between the values of V_s in each case to the full load V_s. The efficiency at any load can be determined from the diagram for that load, by

$$\epsilon = \frac{E_s I_s \cos \phi_s}{E_p I_p \cos \phi_p}.$$

Fig. 78 is not the true diagram of a commercial transformer. For clearness a ratio of 1 to 1 has been portrayed and the losses greatly exaggerated. In practice it will be found impossible to complete the solution graphically because of the extreme flatness of the triangles. The better way is to draw an exaggerated but clear diagram, and obtain the true values of the sides by the algebra of complex imaginary quantities, or if the student is unfamiliar with this method, by the more laborious methods of trigonometry and geometry.

55. Methods of Connecting Transformers. — There are numerous methods of connecting transformers to distributing circuits. The simplest case is that of a single transformer in a single-phase circuit. Fig. 80 shows such an arrangement. This and the succeeding figures have the pressure and current values of the different parts marked on them, assuming in each case a 1 K.W., 1 to 10 step-down transformer. As in Fig. 81, two or more trans-

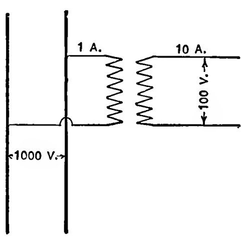


Fig. 80.

formers may have their primaries in parallel on the same circuit, and have their secondaries independent. If the two secondaries of this case are connected properly in series a secondary system of double the potential will result, or by adding a third wire to the point of juncture, as shown by the dotted line of Fig. 82, a three-wire system of distribution can be secured. The secondaries must be connected cumulatively; that is, their instantaneous *E.M.F.*'s must be in the same direction. If connected differentially, there would be no pressure between the two outside sec-

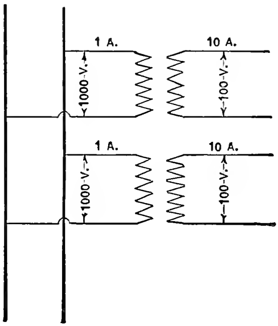


Fig. 81.

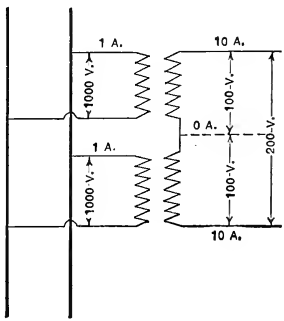


Fig. 82.

ondary wires, the instantaneous pressures of the two coils being equal and opposed throughout the cycle. Again, with the same condition of primaries, the secondaries can be connected in multiple as in Fig. 83. Here the connections must be such that at any instant the *E.M.F.*'s of the secondaries are toward the same distributing wire. The connection of more than two secondaries in series is not common, but where a complex network of secondary distributing mains is fed at various points from a high-tension system, secondaries are necessarily put in multiple.

In many types of modern transformers it is usual to

wind the secondaries (low-tension) in two separate and similar coils, all four ends being brought outside of the case. This allows of connections to two-wire systems of either of two pressures, or for a three-wire system according to Figs. 82 and 83, to be made with the one transformer, this being more economical than using two transformers of half the size, both in first cost and in cost of operation. In many transformers the primary coils are also wound in two parts. In these, however, the four terminals are not always brought outside, but in some cases are led to a porcelain block on which are four screw-connectors and a pair of brass links, allowing the coils to be arranged in series or in multiple according to the pressure of the line to which they are to be connected. From this block two wires run through suitably bushed holes outside the case.

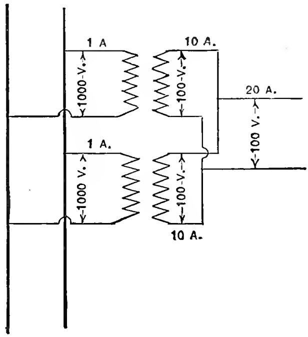


Fig. 83.

A two-phase four-wire system can be considered as two independent single-phase systems, transformation being accomplished by putting similar single-phase transformers in the circuit, one on each phase. If it is desired to tap a two-phase circuit to supply a two-phase three-wire circuit, the arrangement of Fig. 84 is employed. By the reverse connections two-phase three-wire can be transformed to two-phase four-wire. An interesting transformer connec-

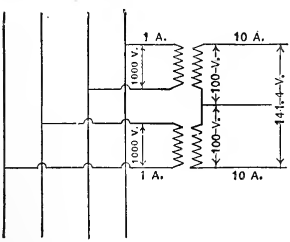


Fig. 84.

tion is that devised by Scott, which permits of transformation from two-phase four-wire to three-phase three-wire. Fig. 85 shows the connections of the two transformers. If one of the transformers has a ratio of 10 to 1 with a tap at the middle point of its secondary coil, the other must have a ratio of 10 to .867 $\left(10 \text{ to } \frac{\sqrt{3}}{2}\right)$. One terminal of the secondary of the latter is connected to the

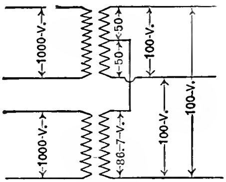


Fig. 85.

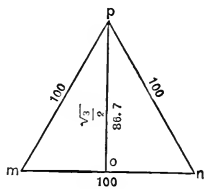


Fig. 86.

middle of the former, the remaining three free terminals being connected respectively to the three-phase wires. In Fig. 86, considering the secondary coils only, let *mn* represent the pressure generated in the first transformer. The pressure in the second transformer is at right angles (§ 5) to that in the first, and because of the manner of connection, proceeds from the center of *mn*. Therefore the line *op* represents in position, direction, and magnitude the pressure generated in the second. From the geometric conditions *mnp* is an equilaterial triangle, and the pressures represented by the three sides are equal and at 60° with the others. This is suitable for supplying a three-phase system. In power transmission plants it is not uncommon to find the generators wound two-phase, and the step-up transformers arranged to feed a three-phase line.

In America it is common to use one transformer for each phase of a three-phase circuit. The three transformers may be connected either Y or Δ. They may be Y on the primary and Δ on the secondary, or *vice versa*. Fig. 87 shows both primary and secondary connected Δ. The pressure on each primary is 1000 volts, and as a 1-K.W. transformer was assumed, i.e., 1 K.W. per phase, there will be one ampere in each, calling for 1.7 ($\sqrt{3}$) amperes in each primary main (§ 33).

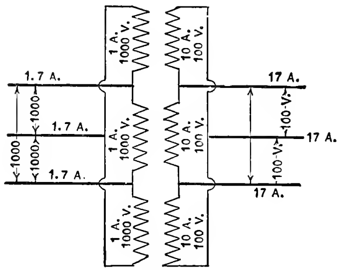


Fig. 87.

This arrangement is most desirable where continuity of service is requisite, for one of the transformers may be cut out and the system still be operative, the remaining transformers each taking up the difference between $\frac{1}{3}$ and $\frac{1}{2}$ the full load; that is, if the system was running at full load, and one transformer was cut out, the other two would be overloaded $16\frac{2}{3}$ per cent. Even if two of them were cut out, service over the remaining phase could be maintained. It is not uncommon to regularly supply motors from three-phase mains by two somewhat larger transformers rather than by three smaller ones.

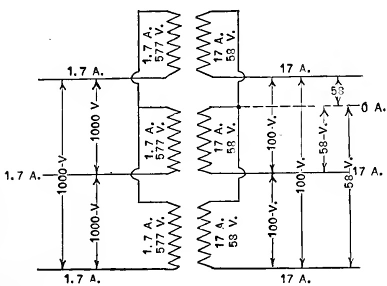


Fig. 88.

Fig. 88 shows the connections for both primaries and secondaries in Y. If in this arrangement one trans-

former be cut out, one wire of the system becomes idle, and only a reduced pressure can be maintained on the remaining phase. The advantage of the star connection lies in the fact that each transformer need be wound for only 57.7 per cent of the line voltage. In high-tension transmission this admits of building the transformers much smaller than would be necessary if they were Δ connected. Fig. 89 shows the connections for primaries in Δ, secondaries in Y; and Fig. 90 those for primaries in Y and secondaries in Δ. By taking advantage of these last two arrangements, it is possible to raise or lower the voltage

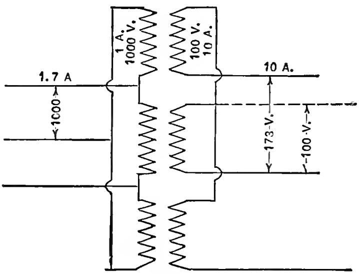


Fig. 89.

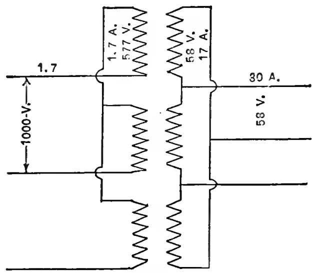


Fig. 90.

with 1 to 1 transformers. With three 1 to 1 transformers, arranged as in Fig. 89, 100 volts can be transformed to 173 volts; while if connected as in Fig. 90, 100 volts can be transformed into 58 volts.

Fig. 91 shows a transformer and another one connected as an autotransformer doing the same work. Since the required ratio of transformation is 1 to 2, the autotransformer does the work of the regular transformer with one-half the first cost, one-half the losses, and one-half the drop in potential (regulation). The only objection to this method of transformation is that the primary and second-

ary circuits are not separate. With the circuits grounded at certain points, there is danger that the insulation of the low-tension circuit may be subjected to the voltage of the high-tension circuit. One coil of an autotransformer must be wound for the lower voltage, and the other coil for the

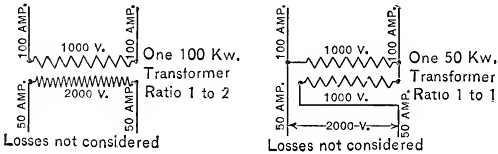


Fig. 91.

difference between the two voltages of transformation. The capacity of an autotransformer is found by multiplying the high-tension current by the difference between the two operative voltages. Autotransformers are often called compensators. Compensators are advantageously used

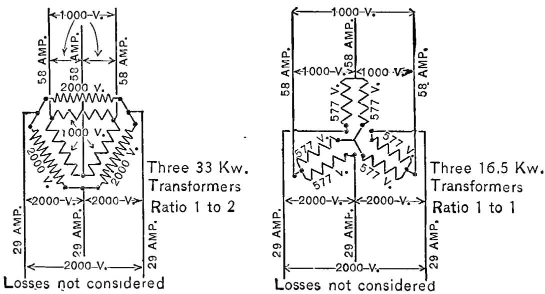


Fig. 92.

where it is desired to raise the potential by a small amount, as in boosting pressure for very long feeders. Fig. 92 shows three 1 to 2 transformers connected in Δ on a three-phase system, and three 1 to 1 compensators connected in Y to do the same work.

From a two-phase circuit, a single-phase *E.M.F.* of any

desired magnitude and any desired phase-angle may be secured by means of suitable transformers, as shown in Fig. 93. Suppose the two phases X and Y of a two-phase system be of 100 volts pressure, and it is desired to obtain a single-phase $E.M.F.$ of 1000 volts and leading the phase X by 30°. As in Fig. 94, draw a line representing the

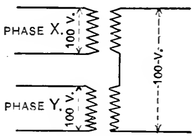


Fig. 93.

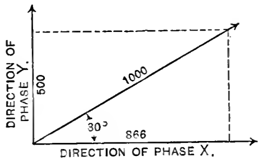


Fig. 94.

direction of phase X. At right angles thereto, draw a line representing the direction of phase Y. From their intersection draw a line 1000 units long, making an angle of 30° with X. It represents in direction and in length the phase and the pressure of the required $E.M.F.$ Resolve this line into components along X and Y, and it becomes evident that the secondary of the transformer connected to X must supply the secondary circuit with 866 volts and that the secondary of the other must supply 500 volts. Therefore the transformer connected to X must step-up 1 to 8.66 and that connected to Y must step-up 1 to 5. If 10 amperes be the full load on the secondary circuit, the first transformer must have a capacity of 8.66 K.W., and the second a capacity of 5 K.W. The load on X and Y is not balanced.

56. Lighting Transformers.—Because of their extensive use on lighting distributing systems, the various manufacturers have to a great extent standardized their lines of lighting transformers. Power transformers are not as yet

well standardized, probably because they are generally used in such large units as to warrant a special design for each case.

The Wagner Electric Mfg. Co.'s "type M" transformer is illustrated in Fig. 95. It is of the shell type of construction, makers using this type claiming for it superiority of regulation and cool running. In the shell type the iron

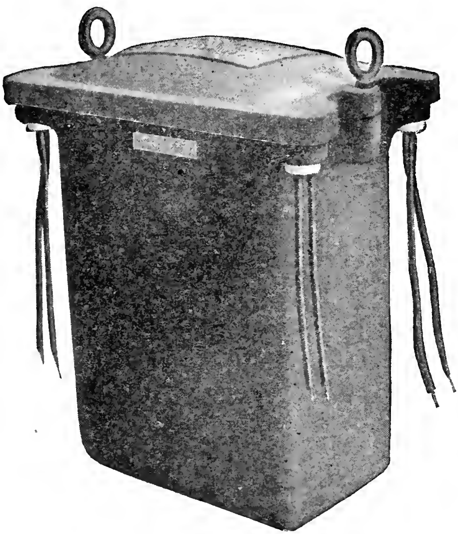


Fig. 95.

is cooler than the rest of the transformer, in the core type it is hotter. As the "ageing" of the iron, or the increase of hysteretic coefficient with time, is believed to be aggravated by heat, this is claimed as a point of superiority of the shell type. However, the prime object in keeping a transformer cool is not to save the iron, but to protect the insulation; and as the core type has less iron and generally less iron loss, the advantages do not seem to be remarkably

in favor of either. In the Wagner "type M" transformers the usual practice of having two sets of primaries and secondaries is followed. Fig. 96 shows the three coils composing one set. A low-tension coil is situated between two high-tensioned coils, this arrangement being conducive to good regulation. The ideal method would be to have the coils still more subdivided and interspersed, but practical reasons prohibit this. Fig. 97 shows the arrangement of the coils in the shell. The space between the coils and the iron is left to facilitate the circulation of the oil in which they are submerged. The laminæ for the shell are stamped each in two parts and assembled with joints staggered. As can be seen from the first cut, all the terminals of the two primary and the two secondary coils are brought outside the case. The smaller sizes of this line of transformers, those under 1.5 K.W., have

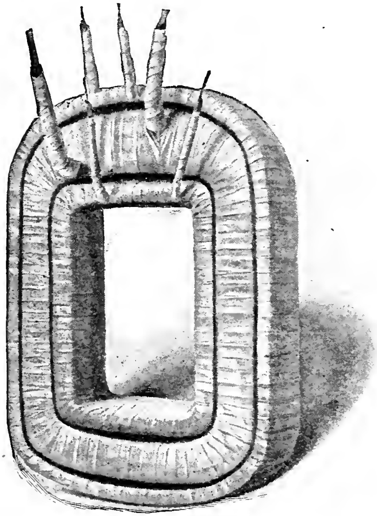


Fig. 96.

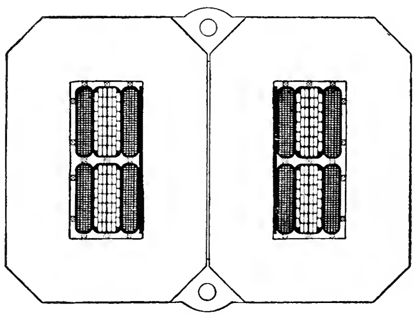


Fig. 97.

sufficient area to allow their running without oil, so the manufacturers are enabled to fill the retaining case with an insulating compound which hardens on cooling.

The General Electric Co.'s "H" transformers are of the core type. In Fig. 76 was shown a sectional view giving a good idea of the arrangement of parts in this type. Fig. 71 is also one of this line of transformers. In it is shown the tablet board of porcelain on which the connections of the two high-tension coils may be changed from series to parallel or *vice versa*, so that only two high-tension wires are brought through the case. Fig. 98 shows the arrangement of the various parts in the assembled apparatus. The makers claim for this type that the coils run cooler because of their being more thoroughly surrounded with oil than those of the shell type. Another point brought forward is that copper is a better conductor of heat than iron; the heat from the inner portions of the apparatus is more readily dissipated than in the shell type. The core has the advantage of being made up of simple rectangular punchings, and the disadvantage of having four instead of two joints in the magnetic circuit. A particular advantage of the "type H" transformer is the ease and certainty with which the primary windings can be separated from the secondary windings. A properly formed seamless cylinder of fiber can be slipped over the inner winding and the outer one wound over it. This is much

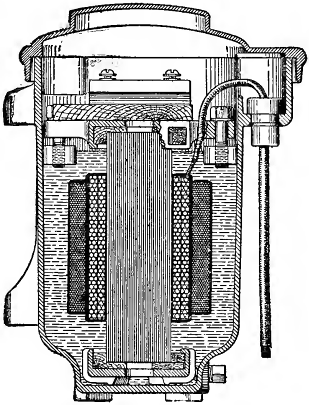


Fig. 98.

more secure than tape or other material that has to be wound on the coils.

Fig. 99.

The Westinghouse "O. D." transformers are of the shell type. The construction of the separate parts is shown in Fig. 99. The coils are wound narrow and to the full depth, and high-tension and low-tension coils alternate side by side instead of from the center out. Fig. 100 shows a 2 K.W. O. D. transformer without the case. A tablet board is used for the terminals of the high-tension coils, but the low-tension wires are all run out of the case. Fig. 101 shows one of the coils. Type O. D. transformers are built from $\frac{1}{4}$ to 25 K.W. for lighting and to 50 K.W. for power. Those of 10 K.W. or less are

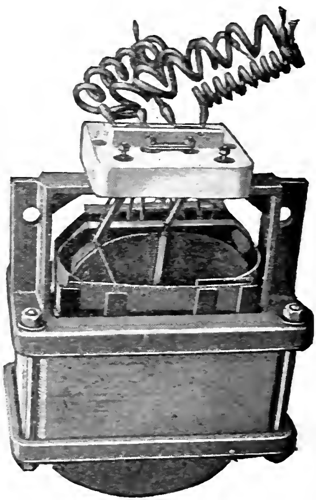


Fig. 100.

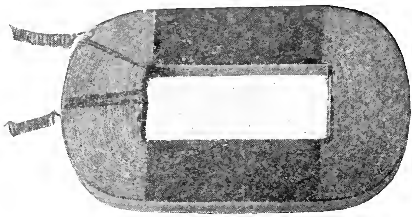


Fig. 101.

in cast-iron cases, those above 10 K.W. in corrugated iron cases with cast tops and bottoms. The corrugations quite

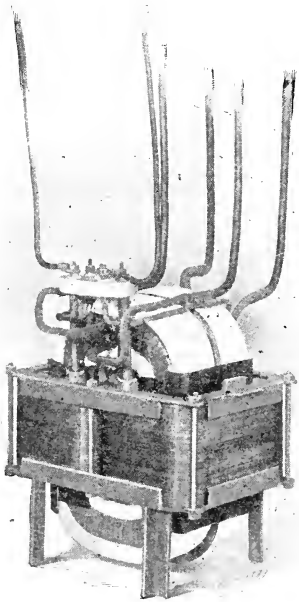


Fig. 102.

materially increase the radiating surface. The windings are submerged in oil.

An example of the Stanley Electric Manufacturing Co.'s

standard line of "type A. O." transformers is given in Fig. 102. These are also of the shell type, with divided primaries and secondaries, four of the eight which belong to a single transformer being shown in Fig. 103.

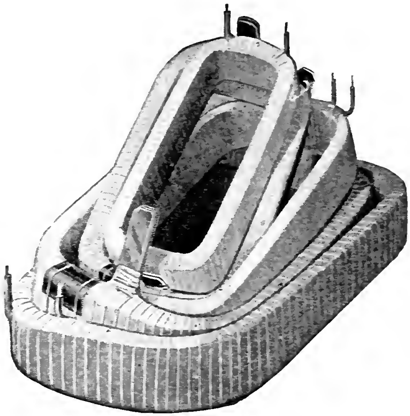


Fig. 103.

57. Cooling of Transformers. — The use of oil to assist in the dissipation of the heat produced during the operations of transformers is almost universal in sizes of less than about 100 K.W., especially if designed for outdoor use. Some small transformers are designed to be self-ventilating, taking air in at the bottom, which goes out at top as a result of being heated. They are not well protected from the weather, and are liable to have the natural draft cut off by the building of insects' nests. Larger transformers that are air cooled and that supply their own draft are used to some extent in central stations and other places where they can be properly protected and attended to. A forced draft is, however, the more common. Where such transformers are employed, there are usually a number

of them; and they are all set up over a large chamber into which air is forced by a blower, as indicated in Fig. 104.

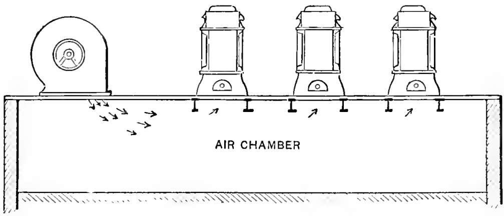


Fig. 104.

Dampers regulate the flow of air through the transformers. They can be adjusted so that each transformer gets its proper share.

Fig. 105 shows a General Electric Company's air-blast transformer in process of construction. The iron core is built up with spaces between the laminæ at intervals; and the coils, which are wound very thin, are assembled in small intermixed groups with air spaces maintained by pieces of insulation between them. The assembled structure is subjected to heavy pressure, and is bound together to prevent the possibility of vibration in the coils due to the periodic tendency to repulsion between the primary and the secondary. These transformers are made in sizes from 100 K.W. to 1000 K.W. and for pressures up to 35,000 volts.

Another method of cooling a large oil transformer is to circulate the oil by means of a pump, passing it through a radiator where it can dissipate its heat. Again cold water is forced through coils of pipe in the transformer case, and it takes up the heat from the oil. There is the slight danger in this method that the pipes may leak and the water may injure the insulation. Water-cooled transformers have been built up to 2000 K.W. capacity.

In those cases where the transformer requires some outside power for the operation of a blower or a pump, the power thus used must be charged against the trans-

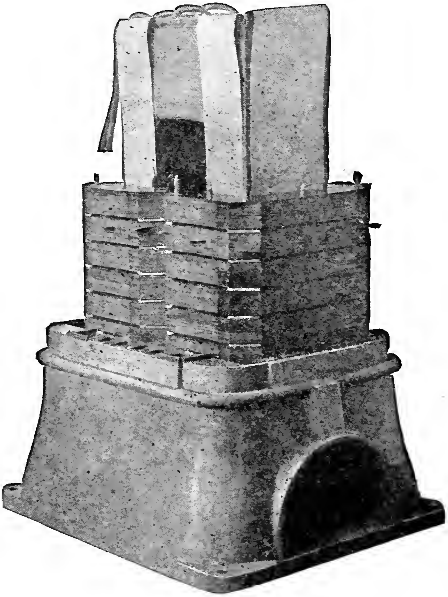


Fig. 105.

former when calculating its efficiency. In general this power will be considerably less than 1% of the transformer capacity.

58. Constant-Current Transformers. — For operating series arc-light circuits from constant potential alternating-current mains, a device called a constant-current transformer is frequently employed. A sketch showing the principle of operation is given in Fig. 106. A primary coil is fixed relative to the core, while a secondary coil is

allowed room to move from a close contact with the primary to a considerable distance from it. This secondary coil is nearly but not entirely counter-balanced. If no current is taken off the secondary that coil rests upon the primary. When, however, a current flows in the two coils there is a repulsion between them. The counterpoise is so adjusted that there is an equilibrium when the current is at the proper value. If the current rises above this value the coil moves farther away, and there is an increased amount of leakage flux. This lowers the *E.M.F.* induced

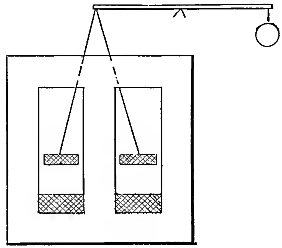


Fig. 106.

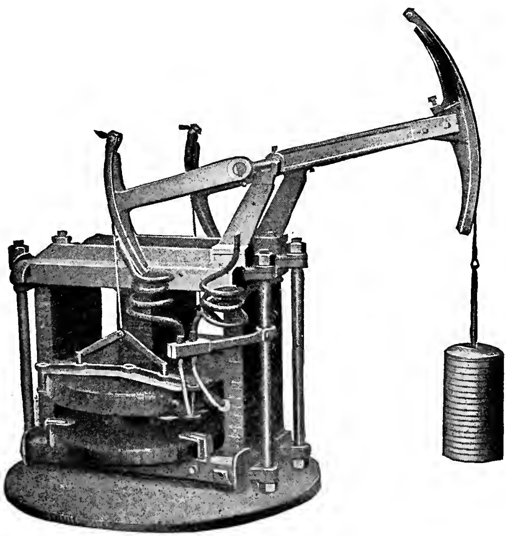


Fig. 107.

in the secondary, and the current falls to its normal value. Thus the transformer automatically delivers a constant current from its secondary when a constant potential is impressed on its primary.

Fig. 107 shows the mechanism of such an apparatus as made by the General Electric Company. The cut is self-explanatory. Care is taken to have the leads to the mov-

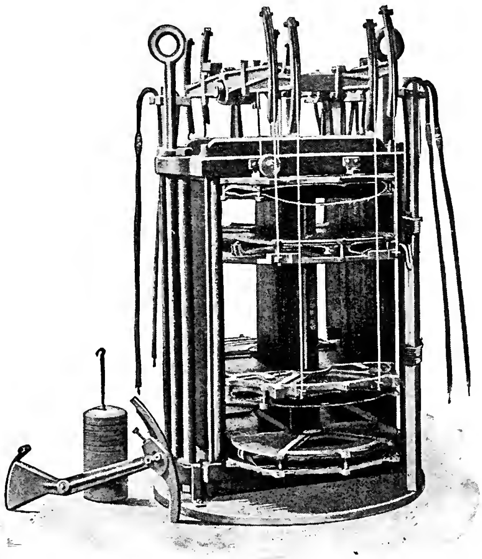


Fig. 108.

ing coil very flexible. Transformers for 50 lamps or more are made with two sets of coils, one primary coil being at the bottom, the other at the top. The moving coils are balanced one against the other, avoiding the necessity of a very heavy counterweight. Fig. 108 shows a 50-light constant-current transformer without its case. Fig. 109 shows a complete 25-lamp apparatus. The tank

is filled with oil, the same as an ordinary transformer. Great care must be taken to keep these transformers level, and to assist in this the larger sizes have spirit-levels built

Fig. 109.

into the case. A pair of these transformers can be specially wound and connected to supply a series arc-light circuit from a three-phase line, keeping a balanced load on the latter.

59. **Design of a Transformer.** — The method of designing a transformer depends upon the specifications as to construction and operation, and upon various values which the designer is forced or sees fit to assume. The following is one method: —

Specifications. — These usually give the capacity in watts, the frequency, the primary voltage, the secondary

voltage, and the conditions of operation, place of installation, whether loaded all day or not, etc.

Assumptions. — The assumption of the following quantities is usually preliminary to any calculation, — the shape of transformer, the current density in the primary, the current density in the secondary, the turns in the primary coil, and the maximum flux density in the iron. The method of design is one of cut and try. A number of values of flux density and various numbers of primary turns are assumed. Efficiency curves are calculated for the various arrangements. The most efficient is ultimately selected; or if none are satisfactory, the course of the design will have brought out the proper direction to take in making new assumptions.

The following design refers to a core-type, step-down, lighting transformer of about 5 K.W. capacity. The assumptions are: 1000 circular mils per ampere in the primary, 1500 circular mils per ampere in the secondary (because this is inside, and has less opportunity of dissi-

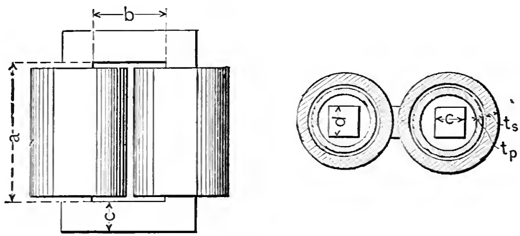


Fig. 110.

pating its heat), 500, 700, and 1000 turns primary successively, and 2000, 3000, and 4000 gausses maximum flux density. The transformer will have the shape shown in Fig. 110. Because of the general use of the English units of measure by most practical mechanics, the dimen-

sions indicated are all expressed in inches. The ratio $\frac{a}{b} = m$ may be conveniently assumed as $m = 1.5$, and the ratio $\frac{c}{d} = n$ is likewise generally made $n = 1$.

I. *To obtain the area*, A, *of the core in square centimeters.*

Let E = impressed primary *E.M.F.*,
$\mathfrak{B}_m$ = assumed maximum flux density,
T_p = assumed number of turns in primary,
and f = frequency.

The instantaneous value of the counter *E.M.F.* of self-induction will be (§ 13, vol. i., § 3)

$$e' = -\frac{T_p d\Phi'}{dt} = -\frac{T_p d(\Phi_m \sin \omega t)}{dt},$$

$$e' = -T_p \Phi_m \omega \cos \omega t,$$

$$e_m = -2\pi f T_p \Phi_m,$$

because the maximum value of the cosine is unity.

$$e = \frac{e_m}{\sqrt{2}} = -\sqrt{2}\,\pi f T_p \Phi_m.$$

At no load this is equal and opposite to the primary impressed pressure, so remembering that

$$\Phi_m = \mathfrak{B}_m A,$$

$$E = -\frac{e}{10^8} = 10^{-8}\sqrt{2}\,\pi f T_p A \mathfrak{B}_m.$$

$$\therefore A = \frac{10^8 E}{1.41\,\pi f T_p \mathfrak{B}_m}.$$

II. *To obtain* c *and* d *in inches.*

$$\frac{c}{d} = n,$$

$$cd = \frac{A}{6.45},$$

$$c = n\frac{\sqrt{A}}{2.54},$$

and
$$d = \frac{1}{n}\,\frac{\sqrt{A}}{2.54}.$$

III. *To obtain the depth of coil winding* t_p *and* t_s *in inches.*

Let d_p = diameter of primary wire, including insulation, in inches.

d_s = diameter of secondary wire, including insulation, in inches,

as found from a wire table; then, allowing $\frac{1}{4}$ inch at each end for insulation,

$$t_p = \frac{\frac{T_p}{2}\,d_p^2}{a - \frac{1}{2}}$$

approximately, since but half the primary is wound on each limb and

$$t_s = \frac{\tau T_p}{2} \cdot \frac{d_s^2}{a - \frac{1}{2}},$$

where τ is the ratio of transformation, $\frac{E_s}{E_p}$.

The value of a is found in the next paragraph.

IV. *To obtain* a *and* b *in inches.* Evidently the transformer could not be assembled unless

$$b > 2\,(t_p + t_s + \text{insulation and clearance}).$$

Assume
$$b = 2\left(t_p + t_s + \frac{7}{8}\right).$$

Now
$$a = mb,$$

and
$$t_s = \tau t_p \left(\frac{d_s}{d_p}\right)^2,$$

so
$$a = 2\, m \left[t_p + \tau t_p \left(\frac{d_s}{d_p}\right)^2 + \frac{7}{8} \right].$$

But also,
$$t_p = \frac{\dfrac{T_p}{2} d_p^{\,2}}{a - \dfrac{1}{2}},$$

so substituting and transposing,

$$\left(a - \frac{7}{4} m\right)\left(a - \frac{1}{2}\right) = m T_p d_p^{\,2} \left[1 + \tau \left(\frac{d_s}{d_p}\right)^2 \right].$$

All the terms of the right-hand member are known, so it may be reduced to a simple number, and set equal to K. Then

$$a^2 - a\left(\frac{7}{4} m + \frac{1}{2}\right) = K - \frac{7}{8} m,$$

and
$$a = \left(\frac{7}{8} m + \frac{1}{4}\right) \pm \sqrt{\left(K - \frac{7}{8} m\right) + \left(\frac{7}{8} m + \frac{1}{4}\right)^2}$$

$$b = \frac{a}{m}.$$

V. *To obtain the volume* v *of iron in cubic centimeters.* — About 90% of a volume occupied by laminated iron is metal.

$$v = 2\,(a + b + 2\,c) \times c \times d \times \overline{2.54}^3 \times 0.9.$$

VI. *To obtain the watts P_h lost in hysteresis.* — According to Steinmetz's Law, using $\eta = .003$,

Hysteresis loss $= .003\, v \mathfrak{B}_m^{\,1.6}$ ergs per cycle.

$$\therefore\ P_h = \frac{.003}{10^7} f v\, \mathfrak{B}_m^{\,1.6}.$$

VII. *To obtain the resistance of the secondary* R_s *in ohms.*—Although surrounding a rectangular core, the coils are usually approximately circular in section, for convenience in winding and in insulating. If the section of the core varies considerably from the square, allowance can be made in estimating the length of a mean turn.

Considering the coil as truly cylindrical, and allowing $\frac{1}{8}$ inch insulation between it and the core, the length of a mean turn

$$l = 2\pi\left(\frac{c}{\sqrt{2}} + \frac{1}{8} + \frac{t_s}{2}\right).$$

The total length of primary wire (both limbs) is then $\tau T_p l$, and its resistance can be found directly in a wire table giving the hot resistances of wires; or, it may be assumed that the transformer will operate at such a temperature that one mil foot has 11 ohms resistance, then

$$R_s = \frac{11\ \tau T_p\ 2\pi\left(\frac{c}{1.41} + \frac{1}{8} + \frac{t_s}{2}\right)}{12 \times \text{circular mils}}.$$

VIII. *To obtain the resistance of the primary* R_p *in ohms.*—Similarly to the above, the length of a mean turn

$$l = 2\pi\left(\frac{c}{\sqrt{2}} + \frac{1}{8} + t_s + \frac{3}{16} + \frac{t_p}{2}\right),$$

allowing $\frac{3}{16}$ inch insulation between the two coils, and the total length of primary wire is $T_p l$.

The resistance can be found in a table, or calculated from

$$R_p = \frac{11\ T_p\ 2\pi\left(\frac{c}{1.41} + \frac{1}{8} + t_s + \frac{3}{16} + \frac{t_p}{2}\right)}{12 \times \text{circular mils}}.$$

IX. *To obtain the foucault current loss P_f in watts.*—Steinmetz has given the empirical formula

$$P_f = 10^{-16}\, v\, (xf\mathfrak{B}_m)^2,$$

where x is the thickness in mils of one lamina. Transformer iron may be assumed to be from 10 to 20 mils in thickness.

X. *To obtain the efficiency at any load, I_s, in per cent.*—

$$\epsilon = \frac{E_s I_s}{E_s I_s + P_h + P_f + I_s^2 R_s + (\tau I_s)^2 R_p}\, 100$$

for a lighting transformer. If the load be inductive the term $E_s I_s$, whenever it occurs, must be multiplied by the power factor ($\cos\phi$). The error involved in the assumption $I_p = \tau I_s$ is negligible.

After calculating the values in each of the preceding steps for the three values of T_p and the three values of $\mathfrak{B}_m$ suggested, the efficiency curve of each transformer should then be drawn, taking points at $\frac{1}{10}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$, and full load. After having selected the most suitable, determine the following values.

XI. *To determine the all-day efficiency in per cent.*—The average lighting transformer is found to be loaded equivalent to full load for 5 hours, and no load for 19 hours, per day. The all-day efficiency is

$$\frac{\textit{watt hours output}}{\textit{watt hours input}}\ \textit{per day}.$$

All day $$\epsilon = \frac{5\, E_s I_s}{5\,[E_s I_s + I_s^2 R_s + (\tau I_s)^2 R_p] + 24\,[P_h + P_f]}\, 100$$

with non-inductive load, I_s, being the full-load secondary current.

XII. *To determine the regulation in per cent.*—In § 53 was shown the method of calculating the magnetic leakage

of this type of transformer. Call the flux linking only the primary coils Φ_p (this is twice that which links the coil of one limb of the transformer). Call that which links only the secondary coils Φ_s. There is practically no voltage drop at no load, so $E_s = \tau E_p$. At full load there is a drop in the primary and in the secondary, due (*a*) to IR drop, (*b*) to self-induction caused by leakage flux. Knowing this leakage flux, by the formula of paragraph I., this section, calculate the voltage drop in primary and in secondary coils, thus,

$$E_{pd} = 10^{-8} \sqrt{2}\, \pi f T_p \Phi_p,$$

and

$$E_{sd} = 10^{-8} \sqrt{2}\, \pi f \tau T_p \Phi_s.$$

The regulation, expressed in per cent, is

$$\text{Regulation} = \frac{E_s - [\tau (E_{pd} + I_p R_p) + E_{sd} + I_s R_s]}{E_s} 100,$$

where $I_p = \tau I_s$, and is the full-load current. Regulation as stated refers to a non-inductive load.

CHAPTER VII.

MOTORS.

60. Rotating Field. — Suppose an iron frame, as in Fig. 111, to be provided with inwardly projecting poles, and that these be divided into three groups, arranged as in the diagram, poles of the same group being marked by the same letter. If the poles of each group be alternately wound in opposite directions, and be connected to a single source of *E.M.F.*, then the resulting current would magnetize the interior faces alternately north and south. If the impressed *E.M.F.* were alternating, then the polarity of each pole would change with each half cycle. If the three groups of windings be connected respectively with the three terminals of a three-phase supply circuit, any three successive poles will assume successively a maximum polarity of the same sign, the interval required to pass from one pole to its neighbor being one-third of the duration of a half cycle. The maximum intensity of either polarity is therefore passed from one pole to the next, and the result is a *rotating field.* If the frequency of the supply *E.M.F.* be f, and if there be p pairs of poles per phase, then the field will

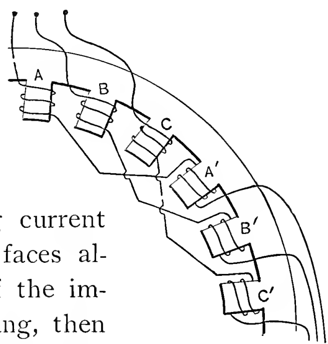


Fig. 111.

make one complete revolution in $\frac{p}{f}$ seconds. It will therefore make $\frac{f}{p} = \frac{V}{60}$ complete revolutions per second. A rotating field can be obtained from any polyphase supply-circuit by making use of appropriate windings.

61. The Induction Motor. — If a suitably mounted hollow conducting cylinder be placed inside a rotating field, it will have currents induced in it, due to the relative motion between it and the field whose flux cuts the surface of the cylinder. The currents in combination with the flux will react, and produce a rotation of the cylinder. As the current is not restrained as to the direction of its path, all of the force exerted between it and the field will not be in a tangential direction so as to be useful in producing rotation. This difficulty can be overcome by slotting the cylinder in a direction parallel with the axis of revolution. Nor will the torque exerted be as great as it would be if the cylinder were mounted upon a laminated iron core. Such a core would furnish a path of low reluctance for the flux between poles of opposite sign. The flux for a given magnetomotive force would thereby be greater, and the torque would be increased.

Induction motors operate according to these principles. The stationary part of an induction motor is called the *stator*, and the moving part is called the *rotor*. It is common practice to produce the rotating field by impressing *E.M.F.* upon the windings of the stator. There are, however, motors whose rotating fields are produced by the currents in the rotor windings.

Fig. 112 shows the stator core and frame of a Westinghouse induction motor, and Fig. 113 shows the same with

the windings in place. Each projection of the core does not necessarily mean a pole; for it is customary to employ a distributed winding, there being several slots per pole

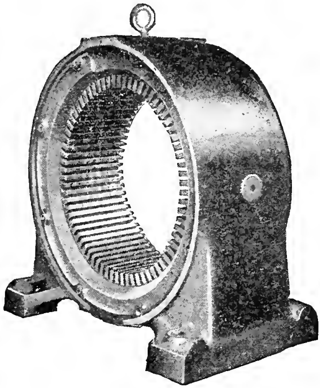


Fig. 112.

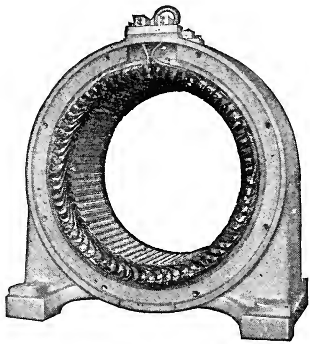


Fig. 113.

per phase. Fig. 114 shows the rotor. The inductors are copper bars embedded in slots in the laminated steel core. They are all connected, in parallel, to copper collars or short-circuiting rings, one at each end of the rotor. They offer but a very small resistance, and the currents induced in them are forced to flow in a direction parallel with the axis. The reaction against the field flux is therefore in a proper direction to be most efficient in producing rotation. A rotor or armature of this type is called a *squirrel cage*.

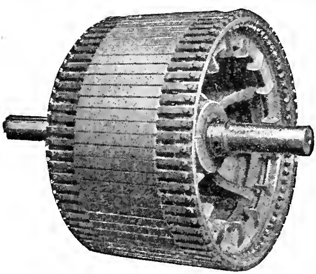


Fig. 114.

62. Principle of Operation of the Induction Motor. — If the speed of rotation of the field be V R. P. M. and that of

the rotor be V' R. P. M., then the relative speed between a given inductor on the rotor and the rotating field will be $V - V'$ R. P. M. The ratio of this speed to that of the field, viz., $\frac{V - V'}{V} = s$, is termed the *slip*, and is generally expressed as a per cent of the synchronous speed. If the flux from a single north pole of the stator be Φ maxwells, then the effective *E.M.F.* induced in a single rotor inductor is $2.22\ p\Phi\ s \frac{V}{60} 10^{-8}$, where p represents the number of pairs of revolving poles. The frequency of this induced *E.M.F.* is different from that of the *E.M.F.* impressed upon the stator. It is s times the latter frequency. The frequency would be zero if the rotor revolved in synchronism with the field, and would be that of the field current if the rotor were stationary. As the slip of modern machines is but a few per cent (2% to 15%), the frequency of the *E.M.F.* in the rotor inductors, under operative conditions, is quite low. The current which will flow in a given inductor of a squirrel-cage rotor is difficult to determine. All the inductors have *E.M.F.*'s in them, which at any instant are of different values, and in some of them the current may flow in opposition to the *E.M.F.* It can be seen, however, that the rotor impedance is very small. As the impedance is dependent upon the frequency, it will be larger when the rotor is at rest than when revolving. It will reduce to the simple resistance when the rotor is revolving in synchronism. Suppose a rotor to be running light without load. It will revolve but slightly slower than the revolving field, so that just enough *E.M.F.* is generated to produce such a current in the rotor inductors that the electrical power is equal to the losses due to friction, windage, and

the core and copper losses of the rotor. If now a mechanical load be applied to the pulley of the rotor, the speed will drop, i.e., the slip will increase. The *E.M.F.* and current in the rotor will increase also, and the rotor will receive additional electrical power, equivalent to the increase in load. The induction motor operates in this respect like a shunt motor on a constant potential direct-current circuit. If the strength of the rotating field, which cuts

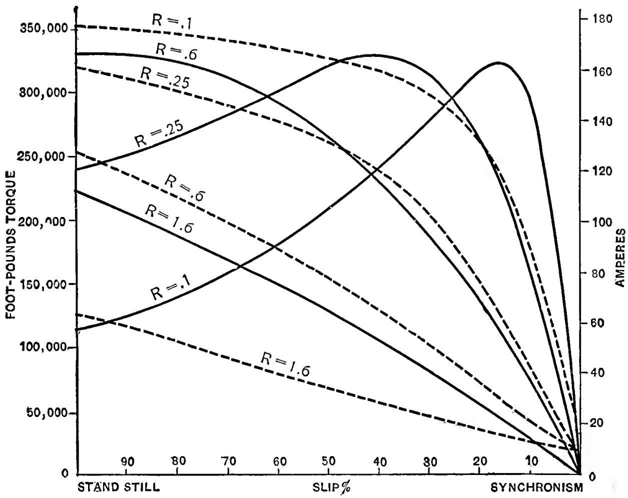


Fig. 115.

the rotor inductors, were maintained constant, the slip, the rotor *E.M.F.*, and the rotor current would vary directly as the mechanical torque exerted. If the rotor resistance were increased, the same torque would require an increase of slip to produce the increased *E.M.F.* necessary to send the same current, but the strict proportionality would be maintained. The rotating magnetism, which cuts the rotor inductors, does not, however, remain constant under vary-

ing loads. As the slip increases, more and more of the stator flux passes between the stator and rotor windings, without linking them. This increase of magnetic leakage is due to the cross magnetizing action of the increased rotor currents. The decrease of linked field flux not only lessens the torque for the same rotor current, but also makes a greater slip necessary to produce the same current. The relation which exists between torque and slip for various rotor resistances is shown in Fig. 115, where the full lines represent torque, and the dotted lines current. An inspection of the curves shows that the maximum torque which a motor can give is the same for different rotor resistances. The speed of the rotor, however, when the motor is exerting this maximum torque, is different for different resistances. This fact is made use of in starting induction motors so that the starting current may not be excessive. Fig. 116 shows a General Electric Form L

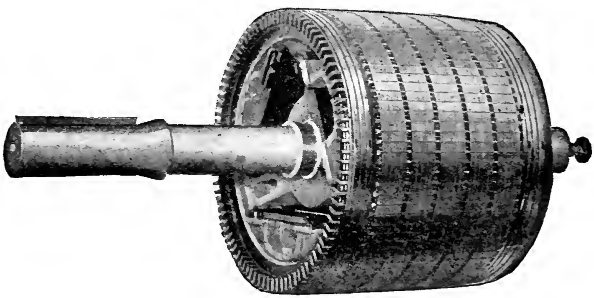


Fig. 116.

rotor. The winding is polar, and not of the squirrel-cage type. The impedance can therefore be easily calculated. The terminals of the windings are connected to a resistance carried on the rotor spider. When the rotor reaches a

proper speed the resistance may be cut out by pushing a knob on the end of the shaft, as shown in diagram. This arrangement permits of a small starting current under load and a large torque. Squirrel-cage motors require several times full-load current to start under load. Fig.

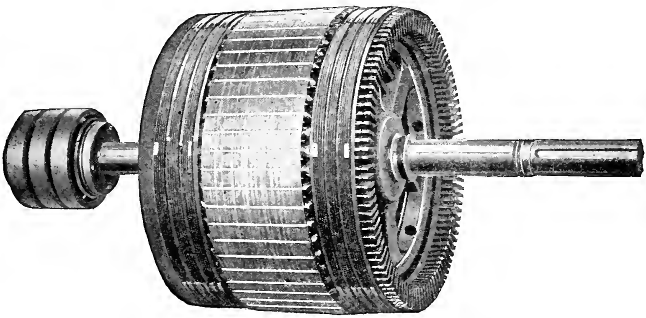


Fig. 117.

117 shows a General Electric Co. form M rotor. The winding is the same as in the Form L, except that its terminals are brought out to three slip-rings. A starting resistance can be placed away from the motor and be connected with the rotor windings by means of brushes rubbing upon the slip-rings.

63. The Transformer Method of Treatment.—It is customary in theoretical discussions to consider the induction motor as a transformer. Evidently when the rotor is stationary the machine is nothing but a transformer, with a magnetic circuit so constructed as to have considerable magnetic leakage. When the rotor is moving, the machine still acts as a transformer; but the ratio of transformation and the frequency of the *E.M.F.* in the rotor, are but s times what they were with a stationary rotor, the mechanical load taking the place of the electric load on the secon-

dary of the transformer. Bearing these facts in mind, the motor may be treated exactly like the transformer. Consider one phase of a polyphase motor. The pressure impressed upon the stator is greater than the pressure which is operative in inducing *E.M.F.* in the rotor. The difference is due to the resistance, the hysteresis, the eddy currents, and the magnetic leakage of the stator. The pressure to overcome each should be subtracted from the impressed pressure in the proper phase relation to get the operative pressure. The equivalent inductance of the magnetic leakage can be calculated for different currents, as was the case in the transformer. The voltage induced in the rotor is $s\tau$ times the operative pressure of the stator where τ is the ratio of transformation. The current which it produces is dependent in magnitude and phase upon the impedance of the rotor windings. From the power represented by this current at the rotor pressure must be subtracted the power lost in resistance, eddy currents, hysteresis, friction, and windage of the rotor. What remains is given out by the motor as useful mechanical power. It should not be forgotten that the frequency of the rotor currents is but s times that of the impressed voltage.

64. Ratio of Transformation. — The ratio of transformation in an induction motor is without appreciable effect upon its operation. For motors of the same capacity it is the practice of the General Electric Company to use the same squirrel-cage rotor for different voltages and different phases. The stator windings alone are altered. Forms L and M rotors are not changed for change of voltage, but must of course be altered for change of phase, as they are polar wound. A certain 4-pole, 3-phase, 60-cycle, 110-

volt, ½-horse-power, General Electric induction motor has 36 slots in the stator, each slot containing 20 conductors of size No. 13. The rotor contains 37 slots, each one containing one No. 2 wire. The slots are staggered by an amount equal to the distance between centers of two consecutive slots. The rotor inductors are connected to short-circuit disks, one on each end of the rotor.

65. Behavior of Induction Motors. — The relations between speed, torque, power factor, efficiency, and current in the case of a typical induction motor operating under normal conditions is represented in Fig. 118.

If the voltage impressed upon an induction motor be increased, there will result a proportional increase in the flux linked with the rotor, and in consequence a propor-

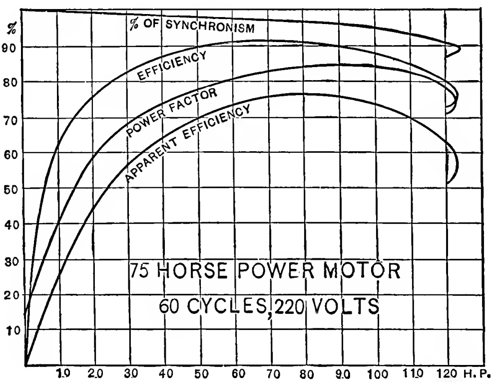


Fig. 118.

tional increase in the rotor current. As the torque depends upon the product of the flux and the rotor ampere turns, it follows that the torque varies as the square of the impressed voltage. The capacity of a motor is therefore

changed when it is operated on circuits of different voltages.

Owing to the low-power factor of induction motors, transformers intended to supply current for their operation should have a higher rated capacity than that of the motors. It is customary to have the kilowatt capacity of the transformer equal to the horse-power capacity of the motor.

The low power-factor is due to magnetic leakage, i.e., flux linked with the stator, but not with the rotor windings. This leakage increases with increase of length of air gap. It is hence desirable to have the gap as small as consistent with mechanical clearance. Concentricity of rotor and stator is to be obtained by making the bearings in the form of end plates fastened to the stator frame. Some makers send wedge gap-gauges with their machines so that a customer may test for eccentricity due to wear of the bearings. A small air gap, besides lowering the leakage and raising the power factor, increases the efficiency and capacity of the motor.

The torque exerted on a constant loaded rotor is continuous and constant in the case of a polyphase motor.

The Stanley Company raise the power factor of their two-phase motors to nearly unity by using condensers to neutralize the lag produced by leakage.

The direction of rotation of a three-phase motor can be changed by transposing the supply connections to any two terminals of the motor. In the case of a two-phase, four-wire motor, the connections to either one of the phases, may be transposed.

66. Starting of Squirrel-Cage Motors. — To avoid the excessive rush of current which would result from connec-

tion of a loaded squirrel-cage motor to a supply circuit, use is made by both the Westinghouse Company and the General Electric Company of starting compensators. These are autotransformers which are connected between the supply mains, and which, through taps, furnish to the motor circuits currents at a lower voltage than that of the supply mains. After the rotor has attained the speed appropriate to the higher voltage, the motor connections are transferred to the mains, and the compensator is thrown

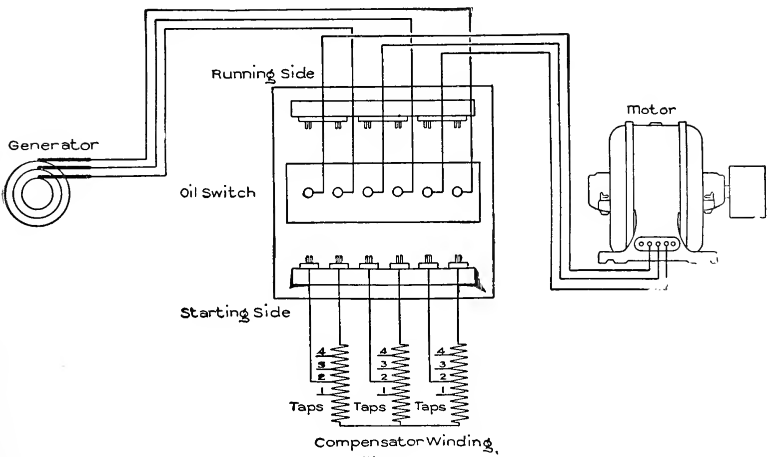


Fig. 119.

out of circuit. The connections are shown in Fig. 119, and the appearance of the General Electric Company compensator is shown in Fig. 120. The change of connections is accomplished by moving the handle shown at the right of the figure. While the compensator is supplied with various taps, only that one which is most suitable for the work is used when once installed. The Westinghouse compensator is shown in Fig. 121. When the handle is down on one side, the autotransformers are in circuit, and

the motor is connected with the low-voltage taps. Upon throwing over the switch the transformers are cut out, and the motor is connected directly with the mains.

Fig. 120.

Where special step-down transformers are used for individual motors, or where several motors are located close to and operated from a bank of transformers, it is sometimes practical to bring out taps from the secondary winding, and use a double-throw motor switch, thereby making provision for starting the motor at low voltage, while avoiding the necessity for a starting compensator.

The General Electric Company make small squirrel-cage motors, with centrifugal friction clutch pulleys; so that although a load may be belted to the motor, it is not applied to the rotor until the latter has reached a certain speed. The starting current is therefore a no-load starting current.

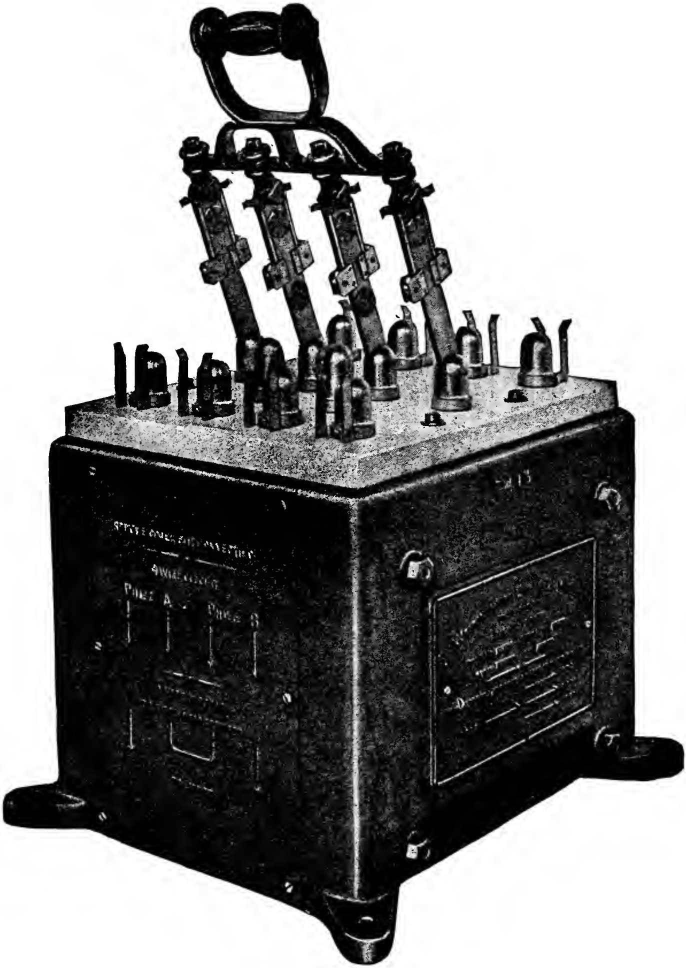


Fig. 121.

67. Phase Splitters. — In order to operate polyphase induction motors upon single-phase circuits, use is made of inductances in series with one motor circuit to produce a

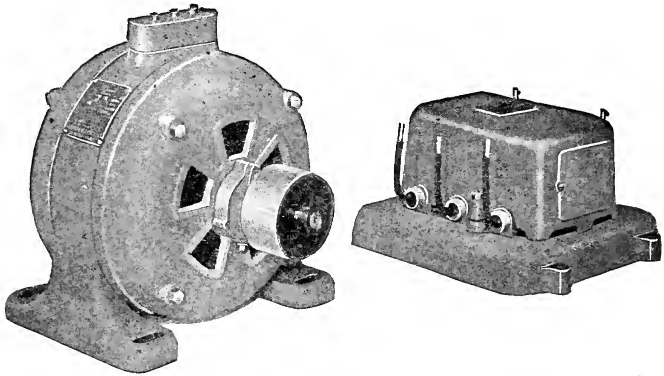


Fig. 122.

lagging current, or of condensers to produce a leading current, or of both — one in each of two legs. The General Electric Company, in its condenser compensator, for use with small motors, as shown in Fig. 122, employs an autotransformer and condenser connected, as in diagram Fig. 123.

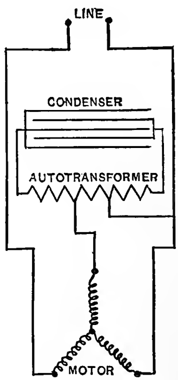


Fig. 123.

The autotransformer is used to step-up the voltage, which is impressed upon the condenser, to 500 volts. The necessary size of the condenser is thereby reduced. The equivalent impedance of the autotransformer and condenser, as connected, is such as to produce a leading current in the one-phase sufficient to give a satisfactory starting torque, and it brings the power factor practically up to unity at all loads.

68. Single-Phase Induction Motors.— A two-phase induction motor will operate fairly well, if, after attaining full

speed, one of the two phases be disconnected from the supply circuit. It will not start from rest under the influence of the one-phase excitation. The load remaining constant, the one phase will take twice its original current. Similarly, a three-phase motor will operate well upon one-phase excitation. The current in this case will be 1.5 times what it previously was. A motor consisting of a rotor-

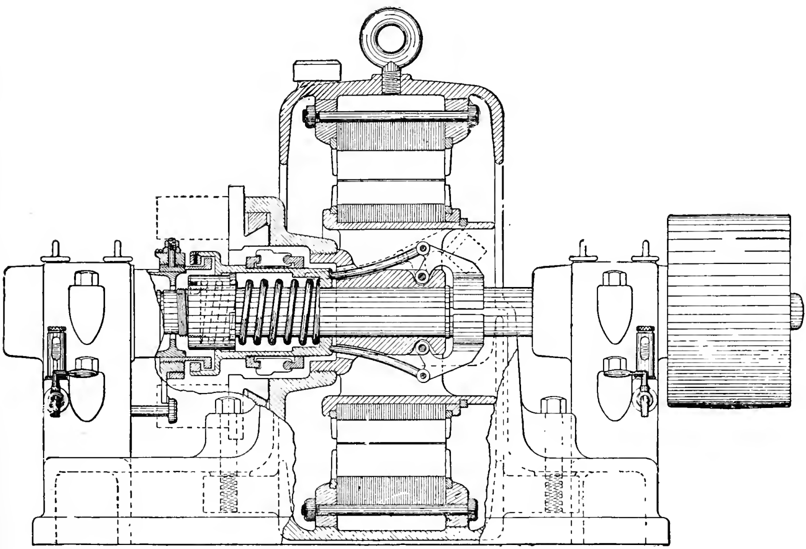


Fig. 124.

and a stator-wound single phase will, in a like manner, operate satisfactorily when once started. In the Wagner single-phase induction motor (Fig. 124), the rotor windings are connected to a commutator, with its brushes joined together by a conductor of low resistance. The stator is supplied with single-phase excitation. The rotor is brought up to speed by the reaction between the current which is induced in the rotor windings and the stator flux.

Upon reaching speed, a centrifugal device, shown in the figure, causes the commutator bars to be short-circuited, and the brushes are simultaneously lifted from the commu-

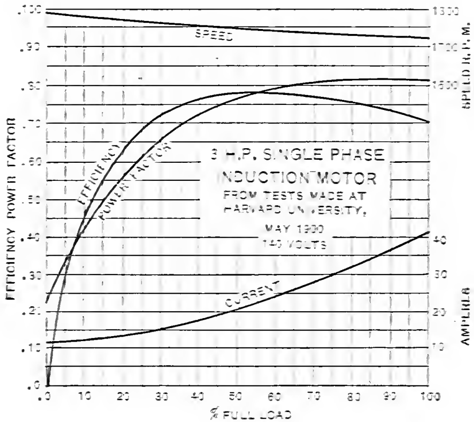


Fig. 125.

tator. Tests have been made upon this type of motor at various universities, including Harvard, University of Illinois, and Purdue University. The results are concordant, and are represented in the curves Fig. 125.

69. The Monocyclic System. — This is a system advocated by the General Electric Company for the use of plants whose load is chiefly lights, but which contains some motors. The monocyclic generator is a modified single-phase alternator. In addition to its regular winding, it has a so-called teazer winding, made of wire of suitable cross section to carry the motor load, and with enough turns to produce a voltage one-fourth that of the regular winding, and lagging 90° in phase behind it. One end of the teazer winding is connected to the middle of the regu-

lar winding, and the other end is connected through a slip-ring to a third line wire.

A three-terminal induction motor is used, the terminals being connected to the line wires either directly or through transformers.

70. Frequency Changers. — These are machines which are used to transform alternating currents of one frequency into those of another frequency. They are commonly used to transform from a low frequency (say from 25 or 40) to a higher one. They depend for their operation upon the variation with slip of the frequency of the rotor *E.M.F.*'s of an induction motor. The common practice for raising the frequency is, to have a synchronous motor turn the rotor of an induction motor in a direction opposite to the direction of rotation of the latter's field. The synchronous motor and the stator windings of the induction motor are connected to the low frequency supply mains. Slip-rings connected to the rotor windings of the induction motor supply current at the higher frequency. The size of the synchronous motor necessary to drive the frequency changer is the same percentage of the total output as the rise of frequency is to the higher frequency.

71. Speed Regulation of Induction Motors. — The speed of an induction motor can be varied by altering the voltage impressed upon the stator, by altering the resistance of the rotor circuit, or by commutating the stator windings so as to alter the multipolarity. The first two methods depend for their operation upon the fact that, inasmuch as the motor torque is proportional to the product of the stator flux and the rotor current, for a given torque the product

must be constant. Lessening the voltage impressed upon the stator lessens the flux, and also the rotor current, if the same speed be maintained. The speed, therefore, drops until enough *E.M.F.* is developed to send sufficient current to produce, in combination with the reduced flux, the equivalent torque. Increasing the resistance of the rotor circuit decreases the rotor current, and requires a drop in speed to restore its value. Both of these methods result in inefficient operation. If the impressed voltage be reduced, the capacity of the motor is reduced. In fact, the capacity varies as the square of the impressed voltage. Changes in the multipolarity of the stator requires complicated commutating devices.

72. Synchronous Motors. — Any excited single-phase or polyphase alternator, if brought up to speed, and if connected with a source of alternating *E.M.F.* of the same frequency and approximately the same pressure, will operate as a motor. The speed of the rotor in revolutions per second will be the quotient of the frequency by the number of pairs of poles. This is called the synchronous speed; and the rotor, when it has this speed, is said to be running in synchronism. This exact speed will be maintained throughout wide ranges of load upon the motor up to several times full-load capacity.

To understand the action of the synchronous motor, suppose it to be supplied with current from a single generator.

Let $E_1 =$ *E.M.F.* of the generator,
$E_2 =$ *E.M.F.* of the motor at the time of connection with the generator,
$\theta =$ Phase angle between E_1 and E_2,

$R =$ Resistance of generator armature, plus that of the connecting wires and of the motor armature, and

$\omega L =$ Reactance of the above.

The resultant *E.M.F.*, E which is operative in sending current through the complete circuit, is found by combining E_1 and E_2 with each other at a phase difference θ, as in Fig. 126.

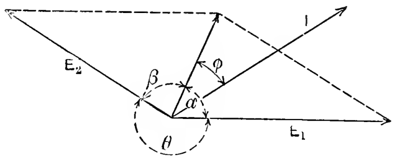


Fig. 126.

Representing the angle between E_1 and E and E_2 and E by α and β respectively, it follows that

$$E = E_1 \cos \alpha + E_2 \cos \beta.$$

This resulting *E.M.F.* sends through the circuit a current whose value is

$$I = \frac{E}{\sqrt{R^2 + \omega^2 L^2}}$$

and it lags behind E by an angle ϕ, such that $\tan \phi = \dfrac{\omega L}{R}$.

The power P_1 which the generator gives to the circuit is

$$P_1 = E_1 I \cos (\alpha - \phi)$$

and the power P_2 which the motor gives to the circuit is

$$P_2 = E_2 I \cos (\beta + \phi).$$

Now, if in either of the above expressions for power, the cosine has any other value than unity, then the power will consist of energy pulsations, there being four pulsations per cycle. The energy is alternately given to and received from the circuit by the machine. If the cosine be positive, the amount of energy in one pulsation, which

is given to the circuit, will exceed the amount in one of the received pulsations. The machine is then acting as a generator. If the cosine be negative the opposite takes place, and the machine operates as a motor. As α and β are but functions of E_1, E_2, and θ, and as these latter are the quantities to be considered in operation, it is desirable to eliminate the former. By a somewhat complicated analytical transformation it can be shown that

$$P_1 = \frac{E_1 E_2}{\sqrt{R^2 + \omega^2 L^2}} \cos(\theta + \phi) + \frac{E_1^2}{\sqrt{R^2 + \omega^2 L^2}} \cos\phi,$$

and $$P_2 = \frac{E_1 E_2}{\sqrt{R^2 + \omega^2 L^2}} \cos(\theta - \phi) + \frac{E_2^2}{\sqrt{R^2 + \omega^2 L^2}} \cos\phi.$$

If there were no losses due to resistance, etc., P_1 would be numerically exactly equal to P_2. Neglecting any losses in the machines, except that due to resistance, the algebraic sum of P_1 and P_2 is equal to RI^2. In order to determine the behavior of a synchronous motor when on a given circuit, use is made of the above formula for power, and each case must be considered by itself. The method of procedure is shown in the next article.

73. Special Case. — Suppose a single-phase synchronous motor, excited so as to generate 2100 volts, to be connected to a generator giving 2200 volts, the total resistance of the circuit being 2 ohms and the reactance 1 ohm. Then the angle ϕ of current lag behind the resultant $E.M.F.$ has a value $\tan\phi = \frac{\omega L}{R} = 0.5$, whence $\phi = 26° \ 34'$. A preliminary calculation, using the formulas of the previous article, shows that both machines act as generators for values of θ between 0° and 120°, and between 240° and 360° approximately.

Calculations of P_1 and P_2 for various values of θ between 120° and 240° have been made, and are embodied in the form of curves in Fig. 127. From an inspection of these

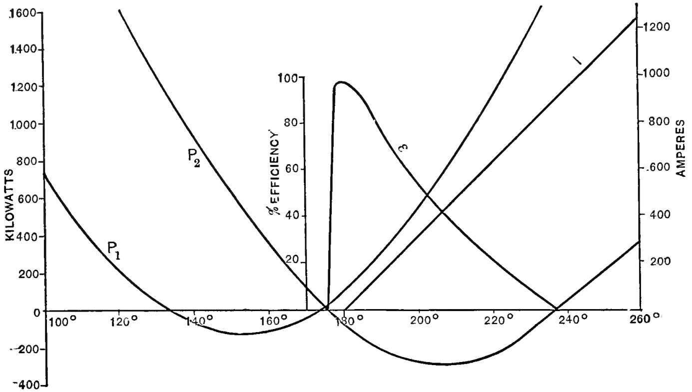


Fig. 127.

curves, and a consideration of the equations from which the curves are derived, the following conclusions may be drawn:—

(*a*) The motor will operate as such for values of θ between 175° and 238°. The difference between these angles may be termed the *operative range.*

(*b*) The generator would operate as a motor for values of θ between 133° and 174°, providing the motor were mechanically driven so as to supply the current and power; i.e., what was previously the motor must now operate as a generator.

(*c*) The motor, within its operative range, can absorb any amount of power between zero and a certain maximum. To vary the amount of received power, the motor has to but slightly shift the phase of its *E.M.F.* in respect to the impressed *E.M.F.*, and then to resume running in

synchronism. The sudden shift of phase under change of load is the fundamental means of power adjustment in the synchronous motor. It corresponds to change of slip in the induction motor, to change of speed in the shunt motor, and to change of magnetomotive force in the transformer.

(*d*) For all values of the received power, except the maximum, there are two values of phase difference θ. At one of these phase differences more current is required for the same power than at the other. The value of the current in either case can be calculated as follows: —

Since $$P_1 + P_2 = RI^2$$

$$I = \sqrt{\frac{P_1 + P_2}{R}}.$$

The values of I are plotted in the diagram. The efficiency of transmission $\epsilon = \frac{P_2}{P_1}$ is also different for the two values of θ. It is also represented by a curve.

If the phase alteration, produced by an added mechanical load on the motor, results in an increase of power received by the motor, the running is said to be *stable*. If on the other hand, the increase of load produces a decrease of absorbed power, the running is unstable.

(*e*) If for any reason the phase difference θ, between the *E.M.F.*'s of the motor and generator, be changed to a value without the operative range for the motor, the motor will cease to receive as much energy from the circuit as it gives back, and it will, therefore, fall out of step. Among the causes which may produce this result are sudden variations in the frequency of the generator, variations in the angular velocity of the generator, or excessive me-

chanical load applied to the motor. In slowing down, all possible values of θ will be successively assumed; and it may happen that the motor armature may receive sufficient energy at some value of θ to check its fall in speed, and restore it to synchronism, or it may come to a standstill.

(*f*) Under varying loads the inertia of the motor armature plays an important part. The shifting from one value of θ to another, which corresponds to a new mechanical load, does not take place instantly. The new value is overreached, and there is an oscillation on both sides of its mean value. This oscillation about the synchronous speed is termed *hunting*. If the armature required no energy to accelerate or retard it, this would not take place.

(*g*) The maximum negative value of P_2—that is, the maximum load that the motor can carry—is evidently when $\cos(\theta - \phi) = -1$ or when $\theta - \phi = 180^\circ$. The formula for the power absorbed by the motor then reduces to

$$P_{2m} = \frac{E_2^2 \cos\phi - E_1 E_2}{\sqrt{R^2 + \omega^2 L^2}} = 320 \text{ K. W.}$$

(*h*) The operative range of the motor can be determined by making P_2 equal to zero. By transformation the formula then becomes

$$\cos(\theta - \phi) = -\frac{E_2^2 \cos\phi}{E_1 E_2}\,.$$

Two values of $(\theta - \phi)$ result, one on each side of 180°. In the case under consideration $\cos(\theta - \phi) = -.851$ and $-\phi = 211^\circ\,40'$ or $148^\circ\,20'$. Since $\phi = 26^\circ\,34'$, $\theta = 238^\circ\,14'$ or $174^\circ\,54'$.

74. The Motor E.M.F. — To determine what value of E_2 will give the maximum value of power to be absorbed by a motor, consider E_2 as a variable in the equation given in (*g*) above.

Differentiating

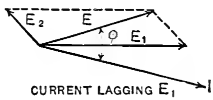
$$\frac{dP_{2m}}{dE_2} = 2\, E_2 \cos\phi - E_1,$$

and setting this equal to zero and solving,

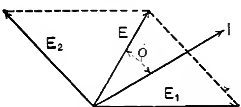
$$E_2 = \frac{E_1}{2\,\cos\phi} = 1230 \text{ volts.}$$

At this voltage the maximum possible intake of the motor is 611 K. W. If the voltage of the motor be above this or below it, its maximum intake will be smaller.

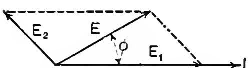
Remembering that the current lags behind the resultant pressure of the generator and motor pressures by an angle ϕ, which is solely dependent upon ω, L, and R, it will be easily seen, from an inspection of Figs. 128, 129, and 130, that the current may be made to lag behind, lead, or be in phase with E_1, by simply altering the value of E_2. This may be done by varying the motor's field excitation. A proper excitation can produce a unit power factor in the transmitting line. The over-excited synchronous motor, therefore, acts like a condenser in producing a leading current, and can be made to neutralize the effect of inductance. The current which is consumed by the motor for a given load accordingly varies with the excitation. The



CURRENT LAGGING E_1
Fig. 128.



CURRENT LEADING E_1
Fig. 129.



CURRENT IN PHASE WITH E_1
Fig. 130.

relations between motor voltage and absorbed current for various loads are shown in Fig. 131.

Synchronous motors are sometimes used for the purpose of regulating the phase relations of transmission lines.

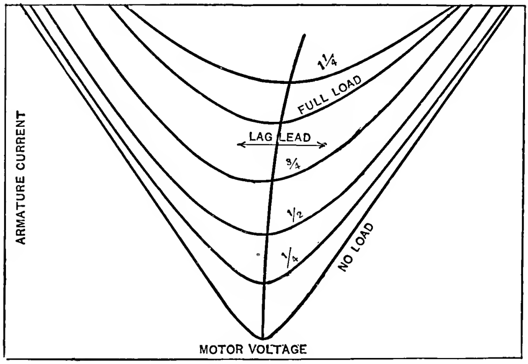


Fig. 131.

The excitation is varied to suit the conditions, and the motor is run without load. Under such circumstances the machines are termed *synchronous compensators*.

The capacity of a synchronous motor is limited by its heating. If it is made to take a leading current in order to adjust the phase of a line current, it cannot carry its full motor load in addition without heating.

75. Polyphase Synchronous Motor.—The discussion which has just been given applies to the single-phase motor. The facts brought out are equally applicable to the polyphase motor. In the latter case each leg or phase is to be considered as a single-phase circuit. The total power is that of each phase multiplied by the number of phases.

76. Starting Synchronous Motors.—These motors do not have sufficient torque at starting to satisfactorily come up

to speed under load. They are, therefore, preferably brought up to synchronous speed by some auxiliary source of power. In the case of polyphase systems an induction motor is very satisfactory. Its capacity need be but $\frac{1}{10}$ that of the large motor. Fig. 132 shows a 750 K. W. quarter-phase General Electric motor with a small induction motor

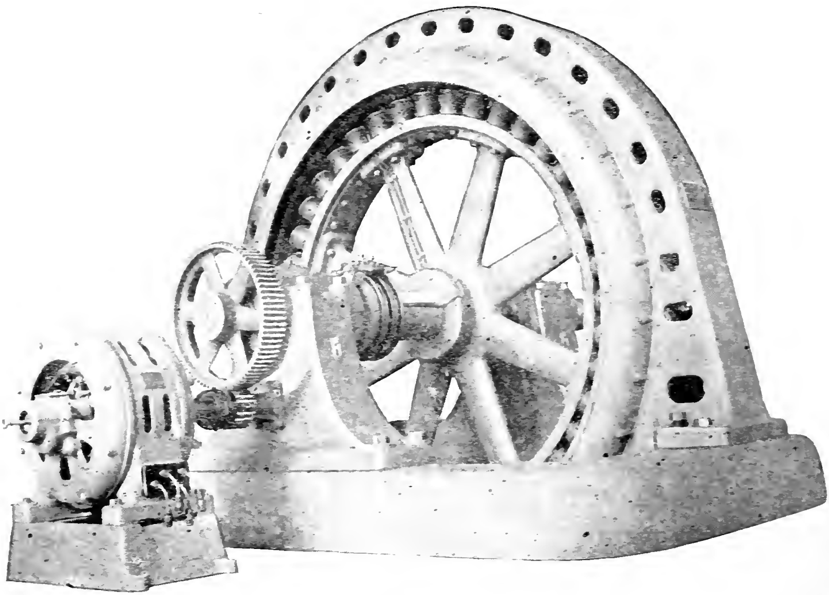


Fig. 132.

geared to the shaft for this purpose. This motor may be mechanically disconnected after synchronism is reached. Before connection of the synchronous motor to the mains it is necessary that the motor should not only be in synchronism, but should have its electromotive force at a difference of phase of about 180° with the impressed pressure. To determine both these points a simple device,

known as a *synchronizer*, is employed. It consists of an incandescent lamp connected in series with the secondaries of two transformers, whose primaries are connected respectively with the line and with the motor terminals. The brightness with which the lamp glows is a measure of the phase difference between the two *E.M.F.*'s. It is customary to so connect the transformers that when the motor *E.M.F.* is at 180° with the line pressure, the lamp will have its greatest brilliancy. As the motor is coming up to speed, the lamp will be alternately bright and dark. The alternations will grow slower as synchronism is approached, and will finally be so slow as to permit the closing of the main switch at the proper instant.

Synchronous motors may be brought up to speed without any auxiliary source of power. The field circuits are left open ; and the armature is connected either to the full pressure of the supply, or to this pressure reduced by means of a starting compensator, such as was described in § 66. The magnetizing effect of the armature ampere turns sets up a flux in the poles sufficient to supply a small starting torque. When, after running a sufficient time as an induction motor, synchronism is nearly attained, the fields may be excited and the motor will come into step. The load is afterwards applied to the motor through friction clutches or other devices. There is great danger of perforating the insulation of the field coils when starting in this manner. This is because of the high voltage produced in them by the varying flux. In such cases each field spool is customarily open-circuited on starting. Switches which are designed to accomplish this purpose are called break-up switches.

77. Parallel Running of Alternators. — Any two alternators adjusted to have the same *E.M.F.*, and the same frequency, may be synchronized and run in parallel. Machines of low armature reaction have large synchronizing power, but may give rise to heavy cross currents, if thrown out of step by accident. The contrary is true of machines having large armature reaction. Cross currents due to differences of wave-form are also reduced by large armature reaction. The electrical load is distributed between the two machines according to the power which is being furnished by the prime movers. This is accomplished, as in the case of the synchronous motor, by a slight shift of phase between the *E.M.F.*'s of the two machines. The difficulties which have been experienced in the parallel running of alternators have almost invariably been due to bad regulation of the speed of the prime mover. Trouble may arise from the electrical side, if the alternators are designed with a large number of poles. Composite wound alternators should have their series compounding coils connected to equalizing bus bars, the same as compound wound direct-current generators.

CHAPTER VIII.

CONVERTERS.

78. **The Converter.** — The converter is a machine having one field, and one armature, the latter being supplied with both a direct-current commutator and alternating-current slip-rings. When brushes, which rub upon the slip-rings, are connected with a source of alternating current of proper voltage, the armature will rotate synchronously, acting the same as the armature of a synchronous motor. While so revolving, direct current can be taken from brushes rubbing upon the commutator. The intake of current from the alternating-current mains is sufficient to supply the direct-current circuit, and to overcome the losses due to resistance, friction, windage, hysteresis, and eddy currents. The windings of a converter armature are closed, and simply those of a direct-current dynamo armature with properly located taps leading to the slip-rings. Each ring must be connected to the armature winding by as many taps as there are pairs of poles in the field. These taps are equidistant from each other.

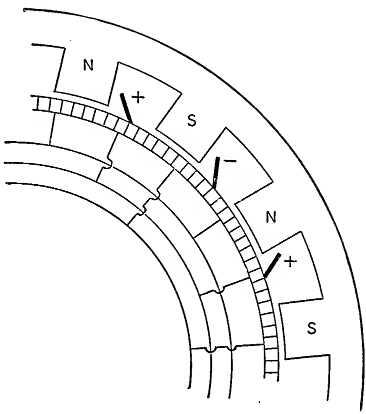


Fig. 133.

There may be any number of rings greater than one. A converter having n rings is called an n-ring converter. The taps to successive rings are $\frac{1}{n}$th of the distance between the centers of two successive north poles from each other. Fig. 133 shows the points of tapping for a 3-ring multipolar converter.

A converter may also be supplied with direct current

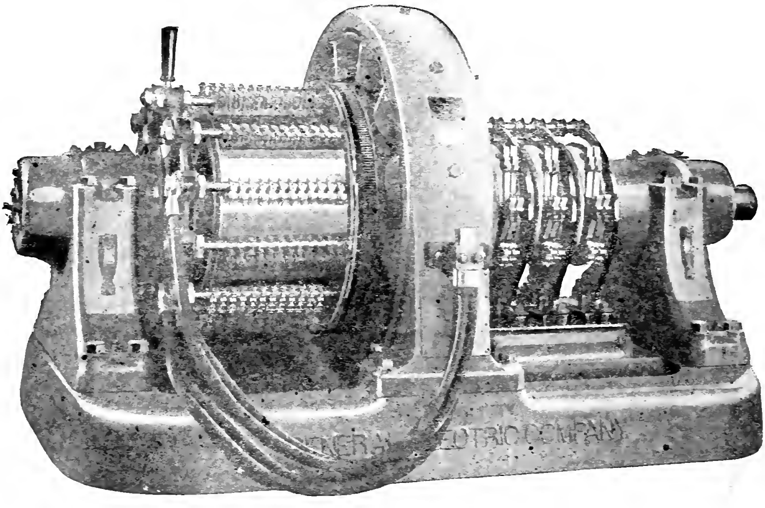


Fig. 134.

through its commutator, while alternating current is taken from the slip-rings. Under these circumstances the machine is termed an *inverted converter*. Converters are much used in lighting and in power plants, sometimes receiving alternating current, and at other times direct current. In large city distributing systems they are often used in connection with storage batteries to charge them

from alternating-current mains during periods of light load, and to give back the energy during the heavy load. They are also used in transforming alternating into direct currents for electrolytic purposes. A three-phase machine for this purpose is shown in Fig. 134.

A converter is sometimes called a rotary converter or simply a rotary.

79. E.M.F. Relations. — In order to determine the relations which exist between the pressures available at the various brushes of a converter,

Let E_d = the voltage between successive direct-current brushes.

E_n = the effective voltage between successive rings of an n-ring converter.

a = the maximum *E.M.F.* in volts generated in a single armature inductor. This will exist when the conductor is under the center of a pole.

b = the number of armature inductors in a unit electrical angle of the periphery. The electrical angle subtended by the centers of two successive poles of the same polarity is considered as 2π

The *E.M.F.* generated in a conductor may be considered as varying as the cosine of the angle of its position relative to a point directly under the center of any north pole, the angles being measured in electrical degrees. At an angle β, Fig. 135, the *E.M.F.* generated in a single inductor G is $a \cos \beta$ volts. In an element $d\beta$ of the periphery of the armature there are $bd\beta$ inductors, each with this *E.M.F.* If connected in series they will yield an *E.M.F.*

of $ab \cos \beta \, d\beta$ volts. The value of ab can be determined if an expression for the $E.M.F.$ between two successive direct-current brushes be determined by integration, and be set equal to this value E_d as follows:

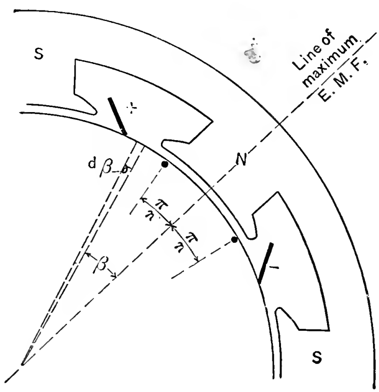


Fig. 135.

$$E_d = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} ab \cos \beta d\beta = 2\ ab.$$

$$\therefore \quad ab = \frac{E_d}{2}.$$

In an n-ring converter, the electrical angular distance between the taps for two successive rings is $\frac{2\pi}{n}$. The maximum $E.M.F.$ will be generated in the coils between the two taps for the successive rings, when the taps are at an equal angular distance from the center of a pole, one on each side of it, as shown in the figure. This maximum $E.M.F.$ is

$$\sqrt{2}\, E_n = \int_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} ab \cos \beta d\beta = 2\ ab \sin \frac{\pi}{n}$$

$$= E_d \sin \frac{\pi}{n}.$$

The effective voltage between the successive rings is therefore

$$E_n = \frac{E_d}{\sqrt{2}} \sin \frac{\pi}{n}.$$

By substituting numerical values in this formula, it is found that the coefficient by which the voltage between

the direct-current brushes must be multiplied in order to get the effective voltage between successive rings is for

2 rings	0.707
3 rings	0.612
4 rings	0.500
6 rings	0.354

In practice there is a slight variation from these co-efficients due to the fact that the air-gap flux is not sinusoidally distributed.

80. Current Relations.— In the following discussion it is assumed that a converter has its field excited so as to cause the alternating currents in the armature inductors to lag 180° behind the alternating *E.M.F.* generated in them.

The armature coils carry currents which vary cyclically with the same frequency as that of the alternating-current supply. They differ widely in wave-form from sine curves. This is because they consist of two currents superposed upon each other. Consider a coil *B*, Fig. 136. It carries a direct current whose value $\frac{I_d}{2}$ is half that carried by one direct-current brush, and it reverses its direction every time that the coil passes under a brush. The coil, as well as all others between two taps for successive slip-rings, also carries an alternating current. This current has its zero

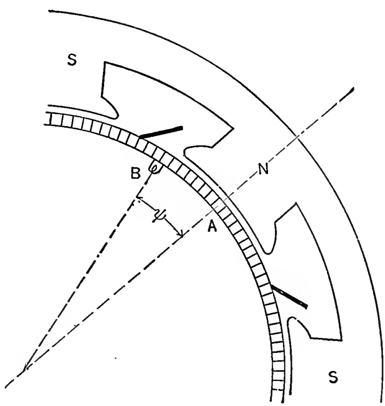


Fig. 136.

value when the point A, which is midway between the successive taps, passes under the brush. The coil being ψ electrical degrees ahead of the point A, the alternating current will pass through zero $\frac{\psi}{2\pi}$ of a cycle later than the direct current. The time relations of the two currents are shown in Fig. 137.

To determine the maximum value of the alternating current consider that, after subtracting the machine losses,

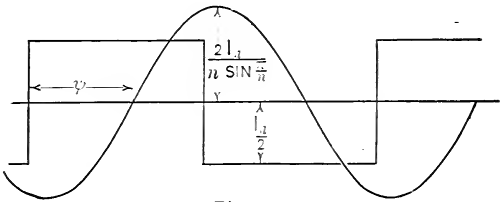


Fig. 137.

the alternating-current power intake is equal to the direct-current power output. Neglecting these losses for the present, if E_n represents the pressure and I_n the effective alternating current in the armature coils between the successive slip-rings, then for the parts of the armature windings covered by each pair of poles

$$E_d I_d = n E_n I_n$$

$$= n \frac{E_d}{\sqrt{2}} \sin\frac{\pi}{n} I_n .$$

Therefore, the maximum value of the alternating current is

$$\sqrt{2}\, I_n = \frac{2 I_d}{n \sin\frac{\pi}{n}} .$$

The time variation of current in the particular coil B is obtained by taking the algebraic sum of the ordinates of

the two curves. This yields the curve shown in Fig. 138. Each inductor has its own wave-shape of current, depending upon its angular distance ψ from the point A.

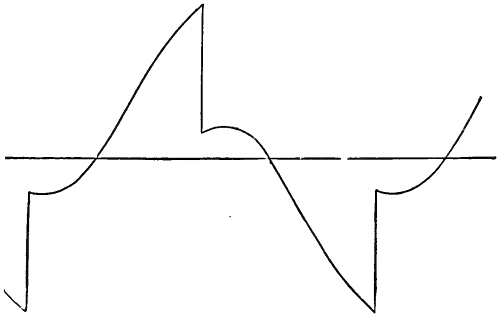


Fig. 138.

81. Heating of the Armature Coils. — The heating effect in an armature coil due to a current of such peculiar wave-shape as that shown in Fig. 138 can be determined either graphically or analytically. The graphic determination requires that a new curve be plotted, whose ordinates shall be equal to the squares of the corresponding current values. The area contained between this new curve and the time axis is then determined by means of a planimeter. The area of one lobe is proportional to the heating value of the current. This value may be determined for each of the coils between two successive taps. An average of these values will give the average heating effect of the currents in all the armature coils. The heating is different in the different coils. It is a maximum for coils at the points of tap to the slip-rings and is a minimum for coils midway between the taps.

The analytical determination is made as follows: For a coil which is ψ electrical degrees from a point midway

between successive slip-ring taps, at the time t seconds after passing a direct-current positive brush, the instantaneous value of the current is

$$I' = \frac{I_d}{2}\left\{\frac{4 \sin (2\,\pi f t - \psi)}{n \sin \frac{\pi}{n}} - 1\right\}.$$

The effective value of the current in this coil is, therefore,

$$I = \sqrt{\frac{1}{\pi}\int_0^{\pi} I'^2\,dt}$$

$$= \frac{I_d}{2}\sqrt{1 - \frac{16 \cos \psi}{\pi n \sin \frac{\pi}{n}} + \frac{8}{n^2 \sin^2 \frac{\pi}{n}}}$$

$$= \frac{I_d}{2}\,Q,$$

where Q represents for simplicity the value of the radical.

The heating, due to the current in this coil, is proportional to $\frac{I_d^2 Q^2}{4}$, and the average heating over the whole armature is proportional to

$$\frac{n}{2\,\pi}\int_{-\frac{\pi}{n}}^{+\frac{\pi}{n}} \frac{I_d^2 Q^2}{4}\,d\psi = \frac{I_d^2}{4}\left(1 - \frac{16}{\pi^2} + \frac{8}{n^2 \sin^2 \frac{\pi}{n}}\right).$$

Inasmuch as the heating of this armature when run as a simple direct-current generator is proportional to $\frac{I_d^2}{4}$, it can, with the same heating, when operating as a converter, put out $\dfrac{1}{\sqrt{1 - \frac{16}{\pi^2} + \frac{8}{n^2 \sin^2 \frac{\pi}{n}}}}$ as much direct current.

82. Capacity of a Converter. — By inserting numerical values in the above equation it is found that a machine has different capacities, based upon the same temperature rise, according to the number of slip-rings, as shown in the following table. The armature is supposed to have a closed-coil winding: —

CONVERTER CAPACITIES.

Used as a	Kilowatt Capacity
Direct-current generator	100
Single-phase converter	85
Three-phase converter	134
Four-phase converter	164
Six-phase converter	196
Twelve-phase converter	227

The overload capacity of a converter is limited by commutator performance and not by heating. As there is but small armature reaction, the limit is much higher than is the case with a direct-current generator.

83. Starting a Converter. — Converters may be started and be brought up to synchronism by the same methods which are employed in the case of synchronous motors. It is preferable, however, that they be started from the direct-current side by the use of storage batteries or other sources of direct current. They may be brought to a little above synchronous speed by means of a starting resistance as in the case of a direct-current shunt motor, and then, after disconnecting and after opening the field circuit, the connections with the alternating-current mains may be made. This will bring it into step.

84. Armature Reaction. — The converter armature currents give rise to reactions which consist of direct-current

generator armature reactions superposed upon synchronous motor armature reactions. It proves best in practice to set the direct-current brushes so as to commutate the current in coils when they are midway between two succes-

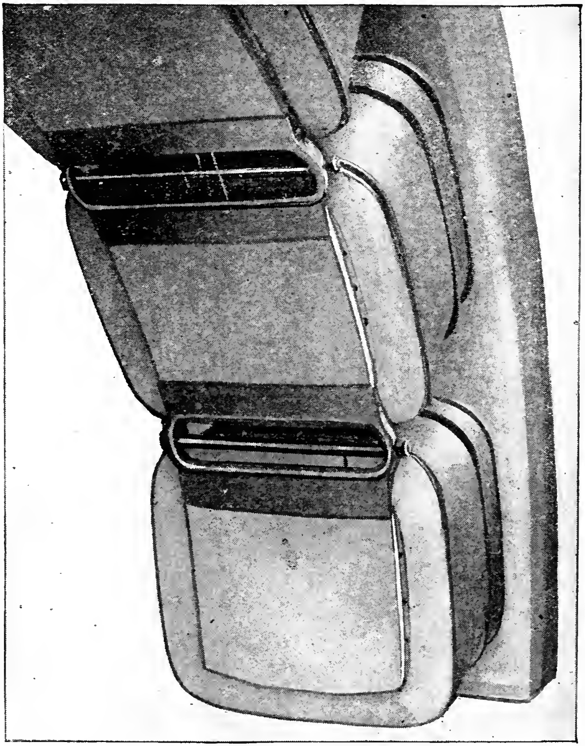


Fig. 139.

sive poles. The direct-current armature reaction, then, consists in a cross-magnetization which tends to twist the field flux in the direction of rotation. When the alternating currents are in phase with the impressed *E.M.F.* they also exert a cross-magnetizing effect which tends to twist the

field flux in the opposite direction. The result of this neutralization is a fairly constant distribution of flux at all loads. Within limits even an unbalanced polyphase converter operates satisfactorily. There is no change of field excitation necessary with changes of load.

The converter is subject to hunting the same as the synchronous motor. As its speed oscillates above and below synchronism, the phase of the armature current, in reference to the impressed *E.M.F.*, changes. This results in a distortion of the field flux, of varying magnitude. This hunting is much reduced by placing heavy copper circuits near the pole horns so as to be cut by the oscillating flux from the two horns of the pole. The shifting of flux induces heavy currents in these circuits which oppose the shifting. Fig. 139 shows copper bridges placed between the poles of a converter for this purpose.

When running as an inverted converter from a direct-current circuit, anything which tends to cause a lag of the alternating current behind its *E.M.F.* is to be avoided. The demagnetization of the field by the lagging current causes the armature to race the same as in the case of an unloaded shunt motor with weakened fields. Converters have been raced to destruction because of the enormous lagging currents due to a short circuit on the alternating-current system.

85. Regulation of Converters. — The field current of a converter is generally taken from the direct-current brushes. By varying this current the power factor of the alternating-current system may be changed. This may vary, through a limited range, the voltage impressed between the slip-rings. As the direct-current voltage

bears to the latter a constant ratio it may also be varied. This is, however, an uneconomical method of regulation. Converters are usually fed through step-down transformers. In such cases there are two common methods of regulation, which vary the voltage supplied to the converter's slip-rings. The first is the method of Stillwell, which is shown in the diagram, Fig. 140.

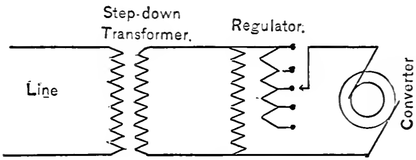


Fig. 140.

The regulator consists of a transformer with a sectional

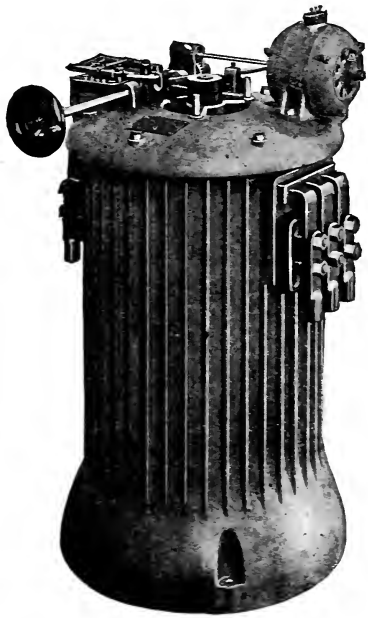


Fig. 141.

secondary. Its ratio of transformation can be altered by moving a contact-arm over blocks connected with the various sections, as shown in the diagram. The primary of the regulator is connected with the secondary terminals of the step-down transformer. The sections of the secondary, which are in use, are connected in series with the step-down secondary and the converter windings.

The second method of regulation is that employed by the General Electric Co. The ratio of transformation of a regulating transformer, which is connected in circuit in the same manner as the Stillwell regulator, is altered by shifting the axes of the primary and secondary coils in respect to each other. Fig. 141 shows such a transformer, the shifting being accomplished by means of a small, direct-current motor mounted upon the regulator. The primary windings are placed in slots on the interior of a laminated iron frame, which has the appearance of the stator of an induction motor. The secondary windings are placed in what corresponds to the slots of the rotor core. The winding is polar; and if the secondary core be rotated by an angle corresponding to the distance between two successive poles, the action of the regulator will change from that of *booster* to that of *crusher*.

CHAPTER IX.

POWER TRANSMISSION.

86. Superiority of Alternating Currents. — The two great sources of energy for use in manufacturing establishments and in land transportation systems are the coal mines and the water powers. While coal can be transported to the point of utilization of the energy, the energy of the waterfall cannot be commercially transmitted to a long distance without the use of electricity. In many cases it is uncertain whether it is not cheaper to transmit the energy of the coal in the form of electrical energy than to transport the coal itself. There is generally greater convenience and greater flexibility in the application and utilization of the transmitted electrical energy.

The electrical transmission can be accomplished by means of direct currents or by means of alternating currents. For transmission over anything but quite short distances the alternating current is preferable to the direct current. Even for short distances, when these pass through densely populated districts, the alternating current is adopted for pure transmission purposes.

The direct current has its points of superiority. Its use is not attended by inductive disturbances with the accompanying drop and sometimes low power factor; it is attended by no appreciable capacity effects; it is not

subject to electric surgings, which sometimes cause insulation perforations, short circuits, and arcing. It permits the use of direct-current motors with their very satisfactory operation as to efficiency, small starting current, overload capacity, and speed control. Its use on transmission lines of over a few miles' length is prohibited by the cost of the line which it necessitates. As will be seen later, long distance electrical power transmission, to be economical or even commercially possible, must be effected by high voltages. Direct-current sparkless commutation is limited to 1000 volts. This limit is dependent upon the economical and mechanical limits of armature peripheral velocity, current density, gap-flux density, and temperature elevation. Furthermore service conditions demand other voltages than those of the transmission line. The direct-current transformer or dynamotor is expensive and not very efficient.

The use of alternating currents is attended by the evil effects of inductance and capacity; the operation of alternating-current motors can be called only fairly satisfactory; but the employment of the very satisfactory, highly efficient, and moderately priced static transformer, makes possible the transmission at high voltages with its accompanying small currents, small line wires, and cheap pole line construction.

The use of the synchronous converter for distribution purposes in connection with alternating-current transmission, constitutes a very satisfactory system, and seems to best meet all the engineering requirements.

87. Frequency. — It is customary to call frequencies above 60 high, and those below 60 low. The proper fre-

quency for a transmission and distributing system is dependent upon a number of variables as follows: —

a. High-frequency transformers are smaller and cost less than those of lower frequency. This is seen by inspection of the formula in article 59, I. For the same voltage and flux density, the product of the iron cross-section and the number of turns varies inversely as the frequency. The cross-section of copper would be the same for the same capacity, irrespective of the frequency.

b. High-frequency generators may be constructed cheaper than those of low frequency. For the same field multipolarity a high frequency is associated with high armature speed, and, therefore, greater output. On the other hand, if an armature be run at the greatest peripheral velocity mechanically permissible, a high frequency necessitates a greater field multipolarity, and, therefore, a greater cost and complexity of construction.

c. High frequencies permit of the satisfactory operation of both arc and incandescent lamps. Arcs do not operate well on any frequencies below 40. The satisfactory operation of incandescent lamps depends upon their voltage and candle-power. Low-voltage lamps have fat filaments of large heat capacity which do not drop in temperature so rapidly as high-voltage thin filaments. The same is true of high candlepower filaments. These lamps may be operated satisfactorily at 25 cycles per second. Standard 110-volt, 16 candle-power lamps, however, fatigue the eye at frequencies under 30 cycles.

d. The inductive line drop, $2\pi fL$, varies directly as the frequency. Its value will be considered later. Being greater for high frequencies, it is then more liable to produce poor regulation at points of distribution.

e. The capacity charging current also varies directly as the frequency.

f. The wattless currents due to inductance and capacity, therefore, increase with the frequency, and thereby lower the operative capacity of the generator, the transformers, and the line. They also lower the efficiency of operation.

g. High frequencies may necessitate so high a field multipolarity that the angular speed variation of the prime mover will prevent the satisfactory paralleling of the generators. For the same reason, the running of synchronous motors and of synchronous converters may be unsatisfactory.

h. Induction motors are best suited for operation on low-frequency circuits. At high frequencies the speed must be high or the motor must be large to avoid running on a low-power factor. The speed could be lowered by increasing the number of poles; i.e., by placing the poles nearer to each other. If the diameter remained the same, this would result in an increase of stator flux leakage, which would reduce the power factor.

88. Voltage.—If the frequency, the amount of transmitted power, and the percentage of power lost in the line, remain constant, the weight of line wire will vary inversely as the square of the voltage impressed upon the line. This depends upon the fact that the cross-section of the wire is not determined by the current density and the limit of temperature elevation, but by the permissible voltage drop. If the impressed voltage on a line be multiplied by n, the drop in the line may be increased n times without altering the line loss. For the line loss is to the total power given to the line as the drop in volts is to

the impressed voltage. To transmit the same power, but $\frac{1}{n}$th the previous current is necessary; and this current, to produce n times the drop, must, therefore, transverse a resistance n^2 times as great as previously.

In transmitting power electrically over long distances, the line cost constitutes a large part of the total invest-

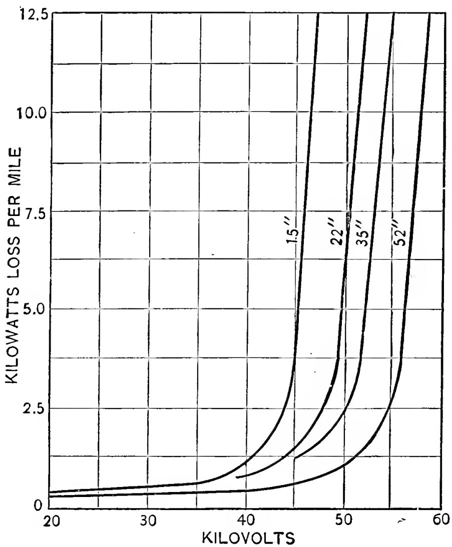


Fig. 142.

ment. In such cases it is desirable to employ as high voltages as possible. There is, however, a limit to the voltage which may be employed. Mr. Charles F. Scott has given some interesting results of experiments carried out on various pole lines. He found that the power lost through the air between wires increased with the impressed voltage, and after a certain voltage was reached, increased very rapidly; that, with a given impressed voltage, the loss decreased as

the distance between the wires was increased; that atmospheric conditions, such as snow, rain, and humidity, had no appreciable effect on the loss; that peaked wave-shaped *E.M.F.*'s gave a greater loss than flat-topped ones; and that the loss decreased as the diameter of the wires was increased. The relations between the distance between wires, the impressed voltage, and the power loss, is shown in Fig. 142.

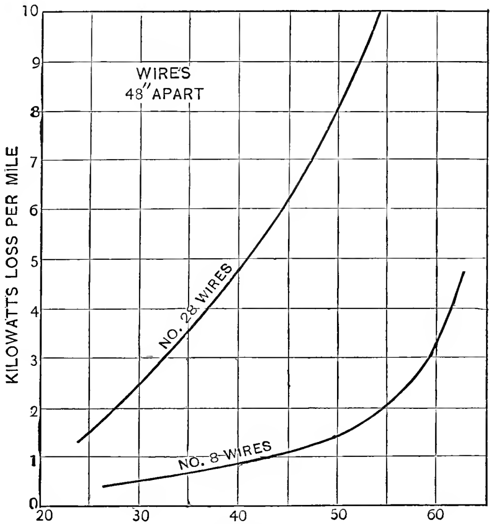


Fig. 143.

The influence of the change of size of wire is shown in Fig. 143, where the distance between the wires was 48 inches in both cases. The influence of the size of the conductor surfaces upon the voltage necessary to break down a dielectric can be illustrated by the apparatus shown in Fig. 144. An applied voltage of sufficient magnitude will produce a spark between pointed conductors, although the

path may be longer than between those which are spherical and are connected in parallel with them.

At high voltages the leakage is accompanied by a hissing sound, and the wires glow visibly at night.

The maximum pressure thus far employed in practice is 60,000 volts. The Standard Electric Company of California uses this voltage on a line constructed of aluminium cable $\frac{7}{8}$ inch in diameter, and made up of 37 strands, the different cables being 42″ from each other.

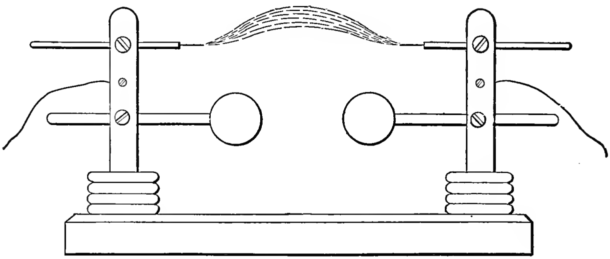


Fig. 144.

Very long distance electrical power transmission is most economically accomplished by the employment of such high voltages. As generators cannot be constructed to give much higher than 13,000 volts, and utilization devices are also limited as to the voltage which may be impressed upon them, step-up and step-down transformers are necessary.

To produce the same percentage loss of power in a line when its length is varied, the impressed voltage must vary as the length. The number of volts per mile vary in practice from 300 to 2000. The choice of voltage is determined by balancing the annual value of the energy lost in the line against the interest and depreciation on the extra capital invested necessary to prevent the loss.

As the distance of transmission decreases there arrives

a point when step-up transformers can be dispensed with and also some step-down transformers. A further decrease of distance permits of transmission and distribution without the use of any transformers.

89. **Number of Phases.** — A comparison of the weights of line wire of a given material, necessary to be used in transmitting a given power, at a given loss, over the same distance, must be based upon equal maximum voltages between the wires. For the losses by leakage, the thickness and cost of insulation, and perhaps the risk of danger to life, are dependent upon the maximum value. A comparison upon this basis gives, according to Steinmetz, the following results: —

Relative weights of line wire to transmit equal power over the same distance at the same loss, with unit power-factor.

2 Wires.	Single-phase	100.0
	Continuous current	50.0
3 Wires.	Three-phase	75.0
	Quarter-phase	145.7
4 Wires.	Quarter-phase	100.0

The continuous current does not come into consideration because of its voltage limitation. The single-phase and 4-wire quarter-phase system each requires one-third more wire than the three-phase system.

By use of the Scott three-phase quarter-phase transformer the transmission system may be three-phase, while the distribution and utilization system may be quarter-phase.

90. **Aluminium Line Wire.** — There are but two materials available for the construction of long-transmission

lines. The high permeability of iron prohibits its use. The remaining materials are copper and aluminium. The prices of both metals vary, and sometimes it is cheaper to use one metal, and again to use the other. A number of aluminium lines have been constructed on the Pacific coast. Not all of them have proved satisfactory. Some of them broke very frequently and without apparent undue strain. Experience has shown that the troubles were due either to improper alloying or impurity of the material, or to improper stringing of the wires. Aluminium has a large temperature coefficient of expansion. Allowance should be made for this. The Standard Electric Co. strings so as to subject their aluminium cables to a strain of 4000 lbs. per square inch at 20° C. Perrine and Baum give the following data concerning a line of commercial aluminium in which they were interested: —

DATA CONCERNING ALUMINIUM.

Size of Aluminium Wire = No. 1 copper.
Resistance of Aluminium Wire = No. 3 copper.
Tensile Strength of Aluminium Wire = No. 5 copper.
Weight of Aluminium Wire = No. 6 copper.

Diameter for the same conductivity 1.270 times copper.
Area " " " " 1.640 times copper.
Tensile Strength for the same conductivity 0.629 times copper.
Weight for the same conductivity 0.501 times copper.

91. Line Resistance. — The resistance of anything but very large lines is the same for alternating currents as for direct currents. In the larger sizes, however, the resistance is greater for the alternating currents. The reason

for the increase is the fact that the current density is not uniform throughout a cross-section of the conductor, but is greater toward its outside. The lack of uniformity of density is due to counter electromotive forces set up, in

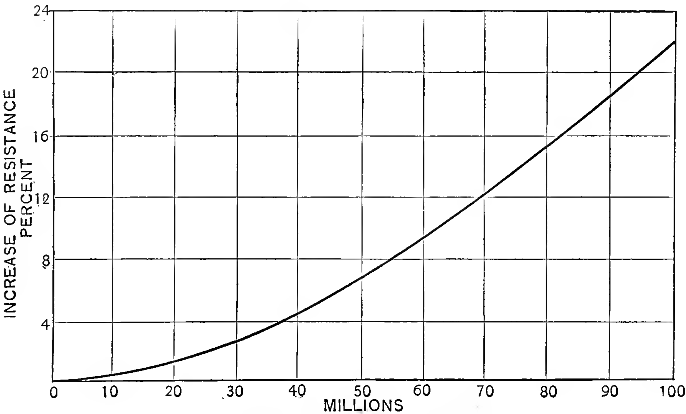


Fig. 145.

the interior of the wire, by the varying flux around the axis of the wire which accompanies the alternations of the current. This phenomena is termed *skin effect*. Its magnitude may be determined from the curve in Fig. 145.

92. Line Inductance. — The varying flux, which is set up between the two-line wires of a single-phase transmission circuit by the current flowing in them, gives rise to a self-induced counter *E.M.F.* The inductance per unit length of single wire is numerically equal to the flux per unit current, which links a unit length of the line. To determine this value consider a single-phase line, with wires of R cms. radius, strung with d cms. between their centers, and carrying a current i. Let a cross-section of

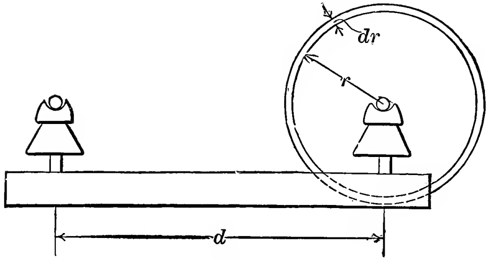


Fig. 146.

the line be represented in Fig. 146. The flux $d\Phi_1$, which passes through an element dr wide and of unit length, is equal to the magnetomotive force divided by the reluctance or

$$d\Phi_1 = \frac{4\,\pi i}{\frac{2\,\pi r}{dr}}.$$

Integrating for values of r between $d - R$ and R

$$\Phi_1 = 2\,i \log\left(\frac{d-R}{R}\right),$$

and practically $$= 2\,i \log\left(\frac{d}{R}\right).$$

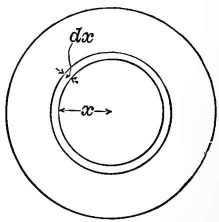


Fig. 147.

There is some flux which surrounds the axis of the right-hand wire, and which lies inside the metal. This is of appreciable magnitude owing to the greater flux density near the wire. Represent the wire by the circle in Fig. 147, and suppose that the current is uniformly distributed over the wire. Then the current inside the circle of diameter x is $\frac{x^2}{R^2}\,i$, and the magnetomotive force, which it produces, is

$$4\,\pi\,\frac{x^2}{R^2}\,i.$$

The flux, however, which it produces, links itself with but $\frac{x^2}{R^2}$ths of the wire. The flux through the element dx, which can be considered as linking the circuit, is therefore

$$d\Phi_2 = \frac{2\,x^3 i dx}{\mu R^4}.$$

Integrating for values of x between 0 and R.

$$\Phi_2 = \frac{2\,i}{\mu 4}.$$

For copper or aluminium wires $\mu = 1$. Hence the total flux linked with the line is

$$\Phi_1 + \Phi_2 = 2\,i\left[\log_\epsilon\left(\frac{d}{R}\right) + \frac{1}{4}\right],$$

and the inductance, in absolute units, being the flux per unit current, is

$$l = 2\log_\epsilon\left(\frac{d}{R}\right) + \frac{1}{2}.$$

This gives by reduction the inductance in henrys per wire per mile as

$$L = \left(80.5 + 740\log\left(\frac{d}{R}\right)\right)10^{-6}.$$

In case of a three-phase line, the inductance in henrys per mile of the whole circuit is

$$L = \left(139 + 1{,}280\log\left(\frac{d}{R}\right)\right)10^{-6}.$$

93. **Line Capacity.** — The two wires of a single-phase transmission line, together with the air between them, act as a condenser. The wires correspond to the condenser plates, and the air to the dielectric. When lines are long, or when the wires are close together, the capacity

LINE CONSTANTS.

(Stanley Electric Manufacturing Company, Pittsfield, Mass.)

Size of Wire B. & S.	Weight.	Diameter.	Area, Circular Mils.	Com'l Resistance.	Inductance Henrys.	Capacity Microfarads 2 Wires.	Charging Cur. of Line, 2 Wires, 60 pps. 10,000 Volts.	Line Constants for Power Transmission, per Mile of Wire.															
								(1) Reactance, X at				(2) Impedance, Z, at				(3) Apparent Resistance, A. P.F. .90.				(3) Apparent Resistance, A. P.F. .80.			
No.	Lbs.	Mils.	C.M.	R.	L.	C.	I. (4)	$f=$ 25	$f=$ 33.3	$f=$ 60	$f=$ 125	$f=$ 25	$f=$ 33.3	$f=$ 60	$f=$ 125	$f=$ 25	$f=$ 33.3	$f=$ 60	$f=$ 125	$f=$ 25	$f=$ 33.3	$f=$ 60	$f=$ 125
0000	3,376	460	211,600	.266	.00148	.0102	.0385	.232	.309	.558	1.16	.353	.408	.618	1.190	.341	.375	.485	.749	.353	.399	.555	.911
000	2,677	410	167,800	.335	.00152	.00996	.0375	.239	.318	.573	1.19	.411	.462	.664	1.236	.407	.442	.554	.826	.412	.460	.613	.984
00	2,123	365	133,100	.422	.00156	.00973	.0366	.245	.326	.588	1.22	.488	.533	.724	1.291	.488	.523	.639	.917	.485	.524	.692	1.073
0	1,685	325	105,500	.533	.00160	.00949	.0358	.251	.334	.603	1.26	.589	.629	.805	1.368	.590	.627	.745	1.034	.577	.628	.789	1.185
1	1,335	289	83,690	.671	.00163	.00926	.0349	.256	.341	.614	1.28	.718	.753	.909	1.445	.716	.754	.874	1.167	.691	.742	.907	1.308
2	1,059	258	66,370	.845	.00165	.00909	.0342	.261	.347	.625	1.30	.884	.914	1.051	1.550	.875	.913	1.036	1.333	.833	.885	1.053	1.459
3	840	229	52,630	1.067	.00170	.00883	.0333	.267	.355	.641	1.33	1.100	1.124	1.245	1.705	1.078	1.117	1.242	1.545	1.015	1.068	1.240	1.655
4	666	204	41,740	1.346	.00173	.00863	.0326	.272	.362	.652	1.36	1.373	1.394	1.496	1.913	1.331	1.370	1.498	1.809	1.241	1.295	1.470	1.897
5	528	182	33,100	1.700	.00177	.00845	.0319	.278	.370	.667	1.39	1.723	1.740	1.826	2.196	1.652	1.693	1.824	2.141	1.528	1.583	1.763	2.197
6	419	162	26,250	2.138	.00181	.00827	.0312	.284	.378	.682	1.42	2.157	2.171	2.244	2.567	2.049	2.091	2.224	2.549	1.882	1.938	2.121	2.565
7	332	144	20,820	2.698	.00184	.00809	.0305	.289	.385	.693	1.44	2.713	2.725	2.780	3.058	2.555	2.598	2.733	3.062	2.332	2.390	2.576	3.025
8	263	128	16,510	3.406	.00188	.00793	.0295	.295	.393	.708	1.48	3.419	3.42[illegible]	3.479	3.714	3.195	3.238	3.377	3.717	2.902	2.962	3.152	3.616
								$f=$ 40	$f=$ 50	$f=$ 66.6	$f=$ 133	$f=$ 40	$f=$ 50	$f=$ 66.6	$f=$ 133	$f=$ 40	$f=$ 50	$f=$ 66.6	$f=$ 133	$f=$ 40	$f=$ 50	$f=$ 66.6	$f=$ 133
0000								.372	.465	.619	1.24	.457	.536	.674	1.268	.403	.444	.512	.785	.437	.493	.586	.960
000								.382	.478	.636	1.27	.508	.584	.719	1.313	.470	.512	.582	.861	.498	.556	.651	1.033
00								.392	.490	.652	1.30	.576	.647	.777	1.367	.552	.596	.667	.952	.574	.633	.730	1.139
0								.402	.502	669	1.34	.667	.739	.855	1.442	.657	.701	.774	1.070	.668	.729	.829	1.234
1								.409	.512	.682	1.36	.785	.844	.956	1.516	.784	.829	.904	1.202	.783	.845	.948	1.356
2								.417	.522	.694	1.39	.942	.993	1.093	1.627	.944	.989	1.066	1.372	.927	.991	1.094	1.514
3								.427	.534	.711	1.42	1.149	1.193	1.282	1.776	1.148	1.195	1.273	1.585	1.111	1.175	1.283	1.709
4								.435	.544	724	1.44	1.415	1.452	1.528	1.971	1.403	1.450	1.530	1.845	1.339	1.425	1.513	1.944
5								.445	.556	.740	1.48	1.757	1.789	1.854	2.254	1.726	1.775	1.856	2.181	1.628	1.695	1.806	2.251
6								.455	.568	.757	1.51	2.186	2.212	2.268	2.618	2.125	2.174	2.257	2.588	1.984	2.052	2.166	2.620
7								.462	.578	.770	1.54	2.737	2.759	2.806	3.107	2.632	2.683	2.767	3.106	2.436	2.506	2.622	3.086
8								.472	.590	786	1.57	3 439	3.457	3.495	3.750	3.273	3.325	3.411	3.756	3.009	3.080	3.198	3.670

(1) $X = 2\pi f L = \dfrac{\text{Wattless } E.M.F. \text{ consumed by line}}{\text{Current in line}}$.

(2) $Z = \sqrt{R^2 + X^2}$.

(3) $A = \dfrac{\text{Drop of voltage in line}}{\text{Current in line}}$; Corresponding to P.F. of .90 or .80. Inductance factor of .44 or .602.

(4) Charging current of line: $I = 2 \times 3.1416 f C E\, 10^{-6}$.
$E = E.M.F.$ $f =$ Frequency.

C = Capacity in Microfarads. L = Inductance in henrys.

Values of X, Z, A per mile of wire.

Values of C and I per mile of circuit.

Distance between wires, 18″.

CURRENT IN MAIN CONDUCTOR. VALUES OF T.

System.	Per Cent Power Factor.				
	100	.95	.90	.85	.80
Single-phase	1.000	1.052	1.111	1.172	1.250
Two-phase (4 wire)	.500	.526	.555	.588	.625
Three-phase (3 wire)	.576	.607	.642	.679	.729

$$\text{Current in main conductors} = \frac{\text{Output in Watts}}{E} \times T.$$

is quite appreciable. The capacity of a two-wire line in microfarads per mile of line is approximately

$$C = \frac{0.04}{\log_\epsilon \left(\frac{d}{R}\right)},$$

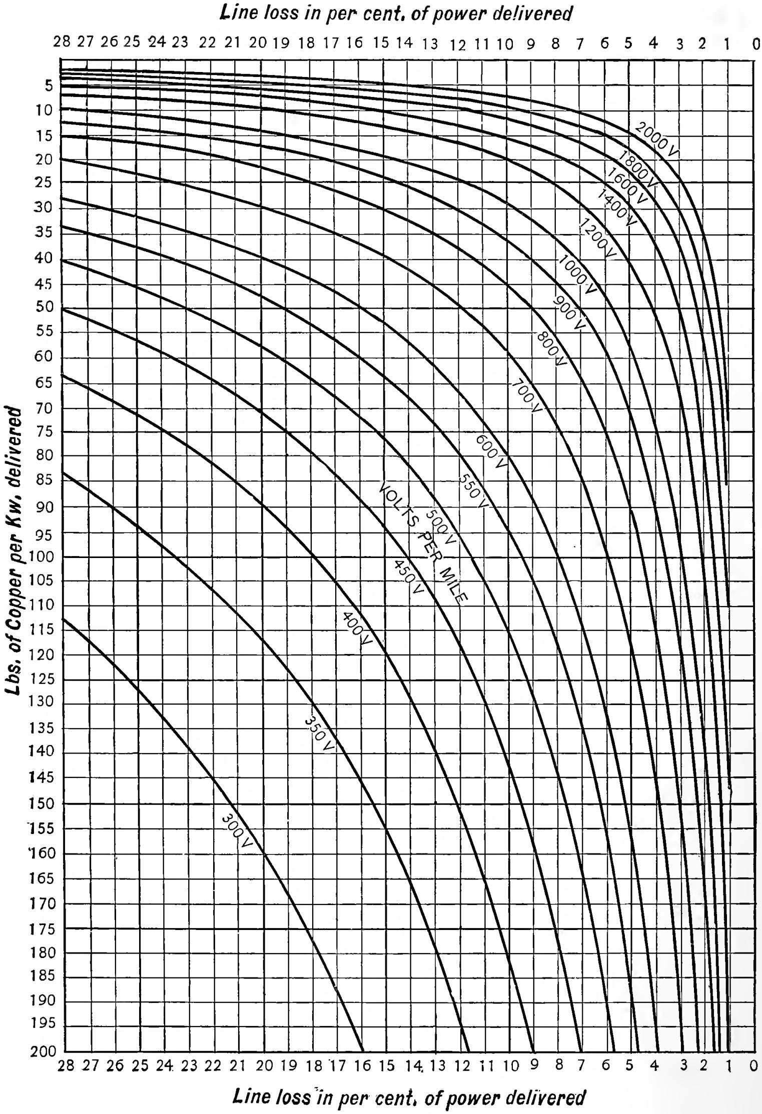
where d is the distance between centers of wires, and R is the radius of the wire, both being measured in the same units.

Because of its capacity, a line which is unloaded takes a current when an alternating $E.M.F.$ is impressed upon it. If the capacity be C microfarads, then E volts at a frequency f would send a charging current (§ 21)

$$I = 2\,\pi f E C\, 10^{-6} \text{ amperes.}$$

94. Line Constants. — The various constants of a transmission line are given in the table on the preceding page.

In calculating the sizes of lines, transformers, and generators of a transmission system, allowance has to be made for the various power factors of the load drawn off at various points. Induction motors, arc lights, and synchronous motors under some excitations have other than unit power-factor. Therefore transformers which supply them must have an excess of capacity sufficient to



Line loss in per cent. of power delivered
28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0
Lbs. of Copper per Kw. delivered
5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150 155 160 165 170 175 180 185 190 195 200
2000 V
1800 V
1600 V
1400 V
1200 V
1000 V
900 V
800 V
700 V
600 V
550 V
500 V
VOLTS PER MILE
450 V
400 V
350 V
300 V
28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0
Line loss in per cent. of power delivered

Fig. 148.

carry the extra current. The line, the step-up transformers, and the generator which supplies the electrical energy, must all have increased capacity. The prime mover, which drives the generator, however, does not need to have this extra capacity. The actual current in all the apparatus being larger than it would be if the power-factor were unity, is accompanied by increased heat losses at every point. The excess of capacity is needed to get rid of this heat, without undue elevation of temperature in the apparatus. The equivalent impedance of the loads and their equivalent power-factor as affecting the line can be determined as shown in the problems of Chapter IV.

95. **Weight of Copper.** — In the curves of Fig. 148 are shown the relations which exist between the transmission loss of power in per cent, the impressed volts per mile, and the weight of copper per K.W. delivered. The loss is expressed as a percentage of the power delivered. The curves apply to a three-phase transmission at unit power-factor. Five per cent has been allowed for sag of lines between poles. To determine the values for aluminium wire, correct by the constants given in § 90.

CHAPTER X.

TESTS.

96. Apparatus.—In the following pages are given directions for a series of experiments designed to give the student dexterity in handling apparatus, a firmer grasp of the phenomena connected with alternating currents, and a knowledge of the methods employed in testing alternating-current apparatus. This course was laid out for use in a laboratory with but a moderate amount of apparatus, and all this apparatus will be here described to avoid the necessity of introducing such descriptions in the directions for the experiments.

The laboratory is supplied with power from an Edison direct-current three-wire system with 117 volts on a side. The largest machine is a 7.5 K.W. double-current generator, which is run as an inverted converter from the Edison current. This is a four-pole machine whose speed can be regulated from 1200 to 1800 R.P.M. This gives a range on the alternating end of 40 ∼ to 60 ∼. There are six slip-rings on the armature, so connected that single-phase current can be had from rings 1 and 4, quarter-phase from 1–4 and 2–6, and three-phase from 1–3–5. The voltage, of course, cannot be altered. A laboratory not supplied with current from a street service could use such a machine, running it as a double-current generator by a steam or gas engine. This would be more desirable than running a regular alternator; as frequently direct current,

as well as alternating, is called for in the experiments. In such case, both frequency and voltage could be regulated. Besides this, there is a 500-watt, 8-pole, 125 ~ alternator belt-driven by a direct-current motor. The wave-shape of this machine was given in Fig. 4. The machine on which most of the tests are run is a double-current generator of about 1 K.W. capacity. This is a bipolar machine fitted with four slip-rings on one end and a commutator on the other. The rings are arranged so that three-phase current is obtained from rings 1–2–3, and single-phase from rings 1–4. This machine serves a multitude of purposes. It can be run as a direct-current motor; as a synchronous motor, either single-phase or three-phase; as a converter, either direct or inverted; and when driven by a belt, as an alternator, single-phase or three-phase; or as a direct-current generator, either shunt wound or separately excited. Its speed can be varied from 1500 to 2400, giving frequencies of 25 ~ to 40 ~. It may be run in parallel with the larger converter when that is slowed down to 40 ~. The equipment of rotating apparatus is completed by two induction motors, one of one-horse power, the other of a half-horse power capacity. They are both wound for three-phase; but the smaller is equipped with a condenser compensator, as described in § 67, and can be run when desired on a single-phase circuit.

The transformer equipment consists of three 1-K.W. 1 to 1 oil-cooled transformers, a half-K.W. 1 to 2 air-cooled transformer, and an old ring-wound armature arranged with taps so that it can serve to transform from 1 to 1, 2, 3, or 4.

For inductive circuits three coils are used. The first, known as Coil 1, was described in § 9. It has about 3000

turns of No. 16 B. & S. wire, 10 ohms resistance, and 0.2 henrys inductance without iron. A bundle of iron wires 16″ long and $1\frac{1}{4}$″ diameter can be inserted in either of the three coils. Coil 2 is in the shape of a hollow cylinder, whose external diameter is $3\frac{1}{4}$″, internal diameter $2\frac{3}{4}$″, and length $3\frac{1}{2}$″. It consists of about 6500 turns of No. 26 B. & S. wire, with an inductance of 0.1 henry and a resistance of 60 ohms. Coil 3 is of the same external appearance as Coil 2, but is made of about 7600 turns of No. 25 B. & S. wire, giving an inductance of 0.141 henry and a resistance of 60 ohms. It will be noticed that Coil 2 and Coil 3 have the same resistance, and that their inductances are as 1 to $\sqrt{2}$. Six paraffined paper condensers of about two microfarads each, are used when condensive circuits are desired.

The instruments used are as follows: Four hot-wire ammeters, with ranges of 1, 3, 15, and 20 amperes respectively. All but the first work across shunts, the small one, however, taking the whole current through its hot wire. These, of course, are used for either alternating or direct currents. Two inclined coil ammeters have ranges respectively of 5 amperes and 50 amperes.

There are three voltmeters, an inclined coil instrument reading to 65 volts; a Cardew hot-wire instrument, reading to 150 volts; and a Weston standard portable voltmeter with two scales, one up to 100 volts, the other up to 200. Any of these may be used on either alternating- or direct-current circuits.

For all the larger measurements a 2.5 K.W. indicating wattmeter is used. For the finer measurements a Weston standard wattmeter, having two scales, is used. The lower scale, for use with pressures of 75 volts or less, reads up

to 75 watts; the upper scale, for use with pressures of 150 volts, reads up to 150 watts. For this instrument a shunt has been constructed, having a coil similar to the current coil of wattmeter, so as to have the same resistance and the same time constant as the latter. This is placed in parallel with the current coil, a small resistance for ballast having first been placed in series with each. The wattmeter then reads up to 300 watts, and is as accurate on inductive as on non-inductive loads.

Certain direct-current instruments are occasionally used, principally a Weston standard portable 150-volt direct-current voltmeter, and a similar five-ampere ammeter. These instruments are used for convenience, and could be dispensed with if necessary.

A means of measuring the rate of rotation of the various machines is essential, and a portable tachometer is by far the best instrument for the purpose. Of course, a greater accuracy can be obtained by using a revolution counter, and noting the number of revolutions in a considerable length of time; but this method is too slow to be satisfactory, and is useless if the speed be fluctuating.

To load a machine electrically, two lamp boards are used. These have each ten key sockets arranged in two rows between three wires. Thus, three wires of a three-wire system may be connected thereto, or the outside wires may be connected together, and all the lamps be put in multiple; and finally, by using the two outside wires only, all the lamps turned on in one row can be put in series with all those on in the other row. Thus a wide range of resistances can be obtained by very small steps, if a few each of 8, 16, 32, 50, and 100 candle-power lamps are in the sockets.

In the following descriptions of experiments, for the sake of brevity, the apparatus needed will not be named; but such notation will be used in the figures, showing the arrangement of apparatus, that the particular apparatus will be indicated. All measuring instruments will be marked with a letter indicating their kind, and a number indicating their capacity; thus A_3 is a three-ampere ammeter, W_{2500} is a 2.5 K.W. wattmeter. In many cases, the manner of drawing will indicate the apparatus, thus:

 is an alternating-current ammeter or voltmeter.

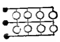
 is a direct-current ammeter or voltmeter.

 is a wattmeter, the binding posts of the current coil being conspicuously large to avoid confusion.

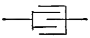
 is a switch designed to shift one ammeter out of circuit and another in without interrupting the continuity of the circuit.

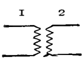
 is a contact-maker, giving a short contact at any desired point in a revolution.

 is a commutator designed to change the direction of current flow in a circuit.

 is a lamp board as described above.

 is an inductive coil.

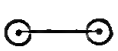
 is a condenser.

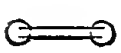


is a transformer, the numbers indicating the relative number of turns.

 is the armature and field coils of a direct-current machine or the direct-current end of a converter.

 is the armature and field coils of an alternating-current machine or the alternating-current end of a converter.

 is to represent a belt-drive between two armatures.

 is to represent a direct connection, or, in the case of a converter, the two ends of the same armature.

97. Exp. 1. Peculiarities of Alternating-Current Circuits. — This experiment consists of some merely qualitative observations calculated to illustrate to the student the difference between alternating currents and the direct currents he has hitherto used.

First Part. — Arrange the apparatus as in Fig. 149, the lamp being by way of protection, in the case of accidental short circuit. Let x be first, the inductive coil known as Coil 1, second, the same with the iron core inserted in it, third a condenser of about 10 M.F. capacity, and fourth a 50-candle power lamp. Apply to these circuits a uniform potential of about 60 volts. Let the frequency be successively 125 ~, 40 ~, and 0 ~, i.e., direct current. With each change note the ammeter reading. It will be observed that with an inductive circuit the current increases as the frequency decreases, and that the maximum current possible flows in the form of direct current. With a condensive circuit the current decreases as the frequency decreases, and is zero with direct current. With a non-reactive circuit, such as the 50 c. p. lamp, the current flow is independent of the frequency.

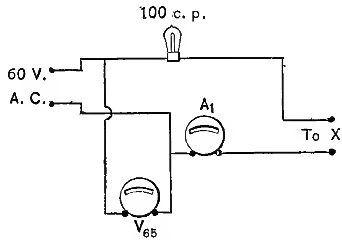


Fig. 149.

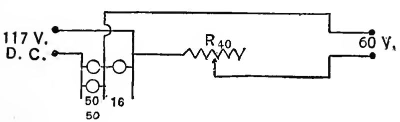


Fig. 150.

Direct current at 60 volts for this experiment can be secured, of course, by running a small dynamo at suitable excitation, but more easily from the 117-volt street service by the arrangement shown in Fig. 150. The lamps can be adjusted to give

60 volts, and the rheostat can take up the difference. This adjustment will be, of course, somewhat different for different loads.

Second Part. — The following solution is one of the many used for blue-prints, and has, besides, the property of turning blue, at the anode only, when a current is passed through it, *if the anode be of iron.* Mix 25 parts (by weight) of ammonium nitrate, NH_4NO_3, and 12.5 parts of ammonium muriate, NH_4Cl. Dissolve 1.3 parts of ferricyanide of potassium, $K_3Fe(CN)_6$, (red prussiate of potash) in 1000 parts of water. Add the ammonium salts. The chemicals should be pure and the water distilled. Keep in a dark place, and use within twenty-four hours.

Prepare an insulating handle, Fig. 151, with three piano-wire projections long enough to be elastic, and whose points may touch a plane surface in a right line and

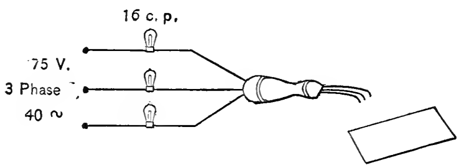


Fig. 151.

near together. Let these wires be connected through 16 c. p. lamps respectively to the terminals of a three-phase system, the pressure being 100 volts or less, and the frequency 40 or less. Lay an uncalendered paper well moistened, but not soaked, in the blue-print solution upon a metal plate, and draw the marking-points quickly across its surface. Blue marks will be left when the current is passing in one of its directions; and these, by their interrupted nature, will show the change of direction in the alternating current. Also the relative displacement or

"stagger" of the rows of marks will show the phase displacements of a three-phase current, as in Fig. 152.

Third Part. — Excite a 16-candle power lamp with alternating current at its rated voltage and a low frequency, say 40 ~. Hold one end of a bar magnet against the bulb and in various positions. The filament will vibrate synchronously with the alternations, due to the regularly recurring attraction and repulsion between the permanent magnetic field of the magnet and the alternating field of the filament. If this experiment fails at first, try varying the frequency, the strength, and polarity of the magnet, and even try other lamps. Often the filament can be made to so vibrate as to touch the glass, and finally rupture itself.

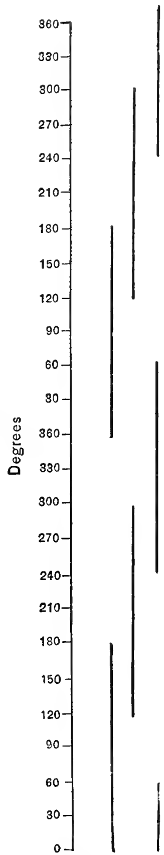


Fig. 152.

98. Exp. 2. Shape of E.M.F. Wave of Alternator. — To perform this experiment use is made of a balance, as shown in Fig. 153. It consists of a hard graphite rod, *C*, of high resistance, through which current is passed from a direct-current constant potential source, two 16 c. p. lamps being in series to guard against accident in case of accidental short circuit. A rolling contact bears upon this rod, and allows of a nice adjustment of the pressure applied to the testing circuit. This pressure can be accurately measured by the standard direct-current voltmeter *V*. In one branch of

the test circuit is placed the telephone receiver, *T*. The operation is as follows: The test circuit, consisting of the armature of the alternator, a lamp or other non-inductive resistance for protection, a contact-maker, and the *E.M.F.* balance just described, is closed for an instant at some point of the revolution which corresponds to some point of the curve of instantaneous pressures. At such instants current will flow through the test circuit, causing the telephone receiver to click sharply; and this click comes with a rapidity corresponding to the rate of revolution of the contact-maker, say 1800 per minute. The sliding contact on the graphite rod is then operated until the continuous direct *E.M.F.* is just equal and opposite to the instantaneous *E.M.F.* put forth by the alternator. Then there will be no flow of current whether the contact-maker be opened or closed, and the receiver will cease to click. The voltage can be read directly on the voltmeter, thus obviating the use of any reduction constants. This method, due to Mershon, is very delicate, since a telephone receiver is sensitive to very small currents.

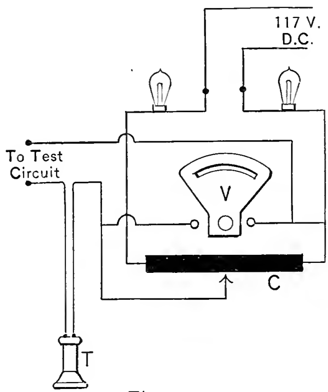


Fig. 153.

To obtain an *E.M.F.* curve from an alternator, arrange the apparatus as in Fig. 154. The contact-maker is connected directly to the shaft of the generator, and is obliged to revolve in unison therewith. Run the alternator at its rated speed and voltage. Set the brush of contact-maker

at the desired beginning point, and balance the instantaneous *E.M.F.* by sliding the balance until no clicking is heard in the receiver. Note the setting of the contact-

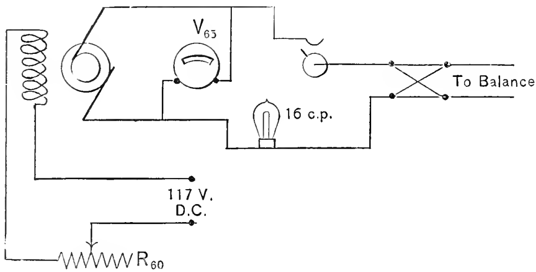


Fig. 154.

maker and the reading of the voltmeter in the balance. Set the contact-maker ahead by five electrical degrees (1 mechanical degree $= p$ electrical degrees, where p is the number of pairs of poles), and repeat as before. Take readings thus by steps of five degrees throughout one complete cycle, i.e., under two poles. Since the instantaneous *E.M.F.* will be in one direction during half a cycle, but in the opposite direction during the other half, and balancing *E.M.F.* is always in the same direction, a commutator must be introduced in the test circuit, as shown. When the commutator is in one position, the voltage readings should be marked +, when in the other position, they should be marked —.

Plot a curve with volts as ordinates and degrees as abscissæ. Indicate on the margin of the curve-sheet the effective value of the curve as obtained from the alternating-current voltmeter. By means of a planimeter measure the area of one lobe of the curve, and find its average ordinate, by dividing the area by the base line, i.e., the

length corresponding to 180°. This may be done in inch and square inch units, if the planimeter be so calibrated, without reducing. Lay this average ordinate off on the margin also. Divide the effective value by the average value to obtain the form-factor (§ 4) of the pressure wave. If this value be about 1.11, the curve is practically a sinusoid.

99. Exp. 3. Shape of Current Wave of Alternator with Inductive Load. — Arrange the apparatus as shown in Fig. 155. The method of procedure is that of Exp. 2.

The instantaneous drop of potential along a non-inductive resistance is proportional to, and in phase with, the current in that resistance. Measure the resistance of

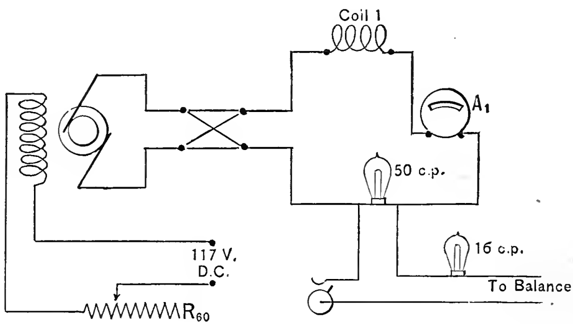


Fig. 155.

the 50 c. p. lamp *under the conditions of use,* since the resistance of a carbon filament varies widely with the temperature; and from this, and the values of instantaneous pressures observed, calculate the instantaneous currents according to Ohm's Law.

Plot a curve with amperes as ordinates, and degrees as abscissæ.

100. Exp. 4. Simultaneous Pressure, Current, and Power Curves from Alternator with an Inductive Load.— Arrange apparatus as in Fig. 156. It will be seen that either a point on the pressure curve (Exp. 2), or a point on the current curve (Exp. 3), can be taken by suitably placing the two-throw switch. Putting the commutator in the main circuit instead of the test circuit is possible when the load is light, does not affect the validity of the observations, and eliminates a possible source of trouble from bad contacts in the test circuit. Readings are to be

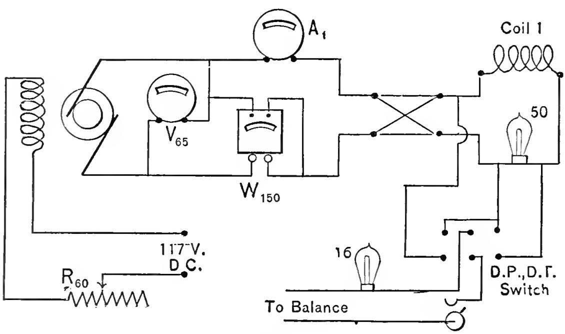


Fig. 156.

taken every five degrees through 400°, a little over one cycle. Take readings for both pressure and current curves each time before moving the contact-maker. This is better than taking a complete current curve, then going back and taking a pressure curve, since there is more liability to distortion due to change in conditions in the latter case.

The voltmeter, ammeter, and wattmeter readings should not vary during the test, and occasional observations should be made to see that this condition is complied with. If it cannot be, readings at stated intervals should be taken, and their averages used in the subsequent calculations.

Plot three curves on one sheet, having degrees as their common abscissæ, and volts, amperes, and watts as their respective ordinates. The instantaneous watts at any abscissa equal the product of the instantaneous volts and amperes for that same abscissa. In general, a separate scale of ordinates will be required for each curve. The curves will have the general relations shown in Fig. 14.

Note the number of degrees intercepted on the axis, between the pressure curve and the current curve. This is the angle of lag, ϕ, the cosine of which is the power-factor of the circuit if the pressure wave is sinusoidal. By the method given in Exp. 2, find the form-factor of the pressure curve. Divide this by the form-factor of a true sinusoid, i.e., 1.11, and call the quotient K. Then $K \cos \phi$ is the power-factor of the curve, whether sinusoidal or not.

By means of a planimeter, measure the area of the lobes of the power curve, being careful to go around the negative part in a counter-clockwise direction. Find the mean ordinate of this curve by dividing the area by the base line, and determine its value in watts by laying off on the scale of ordinates for the power curve.

Fill out the following table, putting in the last column the percentage variation of the individual values from the average.

How Determined.	Watts.	%
By Wattmeter		
By Planimeter		
By $E \times I \times K \cos \phi$		
Average		0

The variations should be within the limits of errors of instruments and observations, say 2%.

101. Exp. 5. Measurement of Self-inductance. — There are various methods of measuring the coefficient of self-induction, two of which are given here. The first is applicable to any series circuit, and consists in the determination of the quantities in the general expression

$$I = \frac{E}{\sqrt{R^2 + (2\,\pi f L)^2}},$$

and solution for L. If E, I, and R be measured respectively in volts, amperes, and ohms, L will be expressed in henrys.

(*a*) Arrange apparatus as shown in Fig. 157, all the lamps being turned off. Insert at x successively Coil 1, Coil 2, and Coil 3. Turn on lamps until a good ammeter deflection is obtained, and note readings of ammeter and voltmeter. The ohmic resistance must in each case be independently determined if not already known.

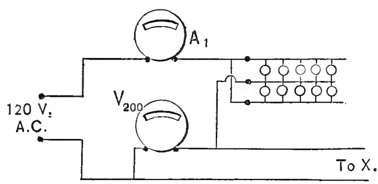


Fig. 157.

Take four sets of observations with each coil; with and without iron core, at 40 ~, and 60 ~.

Solve for the inductance in each case from

$$L = \frac{\sqrt{\left(\frac{E}{I}\right)^2 - R^2}}{2\,\pi f}.$$

Without iron in the magnetic circuit, L is a constant of the circuit, independent of I and f; but when iron is pres-

ent, it varies considerably with I and slightly with f. The variation of inductance with load is the subject of Exp. 7.

Caution must be used in this experiment, that the ammeter be not injured. For instance, the careless removal of an iron core with closed circuit may cause a destructive increase of current.

(*b*) The above method of measuring the inductance is not applicable to branched or parallel circuits with different time constants, for the reason that the resistance of the whole circuit, as measured by direct current, is not the equivalent resistance of the circuit, as explained in § 28. A method using a voltmeter, ammeter, and wattmeter is entirely general, is equally accurate, and does not require

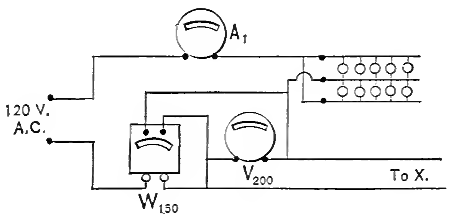


Fig. 158.

the independent determination of the resistance. Arrange the apparatus the same as in the first part of this experiment, with the addition of a wattmeter, as shown in Fig. 158. The method of procedure is the same as before, save that the wattmeter reading is also noted in each case. If I, E, and P be the instrument readings in amperes, volts, and watts respectively, then the inductance in henrys is

$$L = \frac{\frac{E}{I}\sin\left(\cos^{-1}\frac{P}{EI}\right)}{2\pi f}.$$

This equation results from a consideration of the following: —

$$\cos \phi = \frac{P}{EI},$$

$$\phi = \cos^{-1} \frac{P}{EI},$$

$$Z = \frac{E}{I},$$

$$\omega L = Z \sin \phi \text{ (see Fig. 30).}$$

$$\therefore\ L = \frac{\frac{E}{I} \sin \left(\cos^{-1} \frac{P}{EI} \right)}{\omega}.$$

102. Exp. 6. Measurement of Capacity. — When there is no resistance and no inductance in a circuit — as is the case with a condenser — the general formula

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}}$$

reduces to
$$I = \omega C E,$$

hence
$$C = \frac{I}{2\,\pi f E}.$$

Arrange the apparatus as in Fig. 159. Let x be the six condensers taken, first two at a time, then three at a time, then all together, always arranging them in parallel. Note the current and pressure in each case, and solve for C by the above formula. The capacity of any parallel combination of condensers is the sum of the capacities of the component parts, and should so be shown by this experiment. If E and I be in volts and

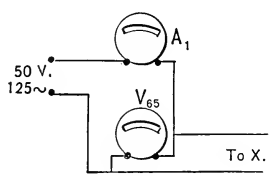


Fig. 159.

amperes respectively, then C will be in farads. In the report reduce these results to microfarads by multiplying by 10^6.

This method is not open to the objections to the similar method of measuring inductance, since here the resistance is practically zero. Yet the second method, using the wattmeter, could be employed. The formula would be

$$\frac{1}{\omega C} = Z \sin\left(\cos^{-1}\frac{P}{EI}\right).$$

But the wattmeter will read zero, since little power is lost in a condenser, so $\phi = 90°$, and

$$\frac{1}{\omega C} = Z,$$

or

$$C = \frac{I}{\omega E},$$

which is the same as deduced from the general formula.

103. Exp. 7. Variation of Coefficient of Self-Induction Under Load. — This experiment may be performed in two parts; (*a*) by varying the magnitude of the measuring current, (*b*) by using a constant measuring current, and varying the saturation of the magnetic circuit by a separate current in a separate winding.

(*a*) Measure the coefficient of self-induction of the fine wire coil of a 1 to 2 transformer by either of the methods of Exp. 5. This current must be made to vary by suitable steps, and this can most easily be done by applying different pressures to the coil. A wide assortment of pressures can be obtained by using different brushes of the converter supplying the energy, and the different steps of the 1, 2, 3, 4 transformer. Determine the value of L for each

of the conditions, and plot a curve having these values as ordinates, and the corresponding currents used in measuring as abscissæ. A curve, such as Fig. 160, will result

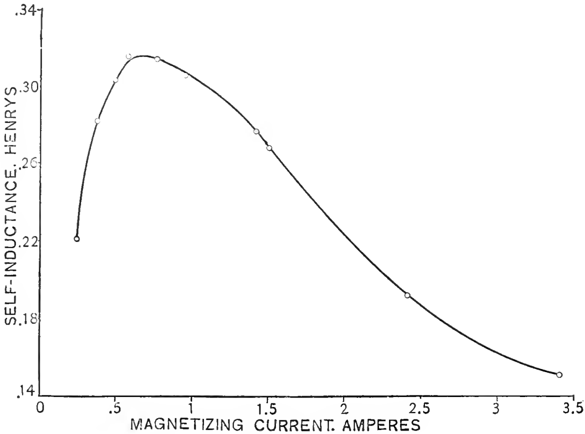


Fig. 160.

with certain irons if the current be started low enough. The sharp rise of the curve at first is due to the fact that at very low densities the permeability increases with density, as is shown in the curves on page 24, Vol. i.

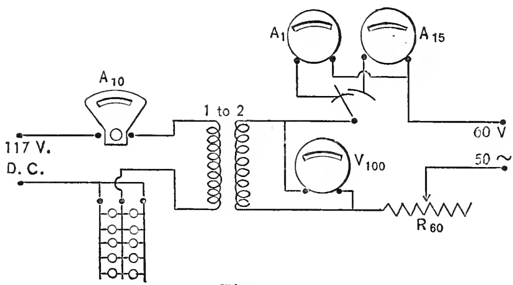


Fig. 161.

(*b*) Arrange the apparatus as in Fig. 161. The measurements of L are made on the fine-wire side of the 1 : 2

transformer, while the permeability is altered by direct current in the low-pressure side. The measuring current should be kept constant; and as it has a tendency to rise as L decreases, resistance will have to be inserted in the alternating-current circuit by adjusting R_{60}. Take readings at suitable steps from zero amperes direct current to the maximum safe temporary ampere capacity of the coil in question, say 15 amperes for a ½ K. W. 55-volt coil.

Calculate the value of L for each of the steps, and plot a curve, using these values as ordinates and the direct-current magnetizing amperes corresponding thereto as abscissæ.

104. Exp. 8. Measurement of Mutual Induction. — Arrange the apparatus as in Fig. 162, the requisite pres-

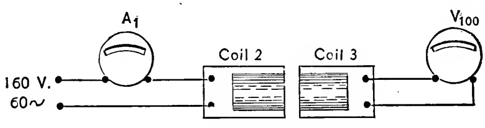


Fig. 162.

sure being secured by stepping up in the 1 : 2 transformer. The experiment consists of three parts: —

(*a*) Determine the coefficient of mutual induction between the two coils from the formula

$$E_3 = \omega M I_2.$$

Transpose Coil 2 and Coil 3, and determine M again, the formula changing, of course, to

$$E_2 = \omega M I_3.$$

The results should be alike if the same current flows in each case.

Finally calculate the theoretical value of M on the assumption of no magnetic leakage from

$$M = \sqrt{L_2 L_3}.$$

L_2 and L_3 were determined in Exp. 5. If the same measuring currents be used throughout, this last value of M will be somewhat above the others, since there is some leakage flux.

(*b*) With the arrangement of Fig. 162 place the iron core with its end flush with the outside of Coil 2, and projecting clear through Coil 3. Move Coil 3 by steps of 2 cm. each from 0 to 24 cm. from Coil 2, and measure the value of M for each step. Be careful that the iron core be not moved relatively to Coil 2.

Plot a curve with centimeters as abscissæ and values of M as ordinates.

(*c*) Repeat the last, keeping the iron flush with Coil 3 however, and moving Coil 2. In this case the current in Coil 2 will vary, and the curve will be distorted by the effects of load and saturation as investigated in Exp. 7.

Plot curve of (*b*) and that of (*c*) on the same sheet and to the same scale.

Be careful not to remove the iron core entirely from the coil that is carrying the current, or the current will exceed the capacity of the ammeter.

105. Exp. 9. Measurement of Power in a Single-phase Inductive Circuit. — There are various ways of measuring power in alternating-current circuits besides using a wattmeter, but none are as satisfactory.

In the following it is desired to measure the power in Coil 1 :—

(*a*) By the three-voltmeter method. Arrange the appa-

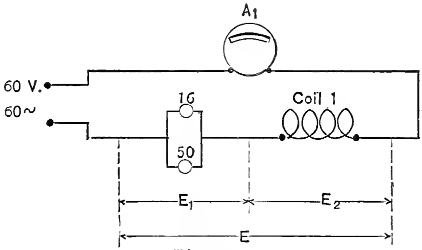


Fig. 163.

ratus as in Fig. 163, the non-inductive lamp resistance, R, having been previously determined. With a 100-volt alternating-current voltmeter note the pressures indicated. The power, P, in the coil is

$$P = \frac{1}{2R}(E^2 - E_1^2 - E_2^2).$$

(*b*) By the three-ammeter method. Arrange the apparatus as in Fig. 164. If I be the reading of A_5, I_1 that of A_3, and I_2 that of A_1, the power in the circuit is

$$P = \frac{R}{2}(I^2 - I_1^2 - I_2^2),$$

where R is the non-inductive resistance of the lamps, which must be independently determined for the conditions of operation.

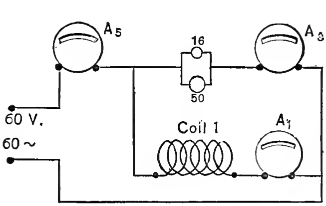


Fig. 164.

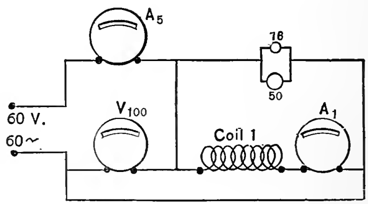


Fig. 165.

(*c*) By the combined method. Arrange apparatus as in Fig. 165. If I be the reading of A_5, I_2 the reading of A_1, and E the reading of the voltmeter, then the power in the coil is

$$P = \frac{R}{2}\left[I^2 - \left(\frac{E}{R}\right)^2 - I_2^2\right],$$

If it be desired to compare the results of (*a*), (*b*), and (*c*), arrangements must be made so that the same difference of potential may be applied to the terminals of Coil 1 in each case.

These methods are rather impractical, and open to the two serious objections that a small error of observation may lead to a serious error in the result, and that the maximum accuracy can only be obtained when about as much power is consumed in the auxiliary devices as in the circuit under test.

106. Exp. 10. Measurement of Power in Polyphase Circuits by Indicating Wattmeters.—In any two-phase circuit of four wires the load can be measured by two wattmeters, one connected regularly in each phase. The sum of their readings is the power in the circuit. In a two-phase four-wire system with a balanced load, one of the wattmeters may be dispensed with, and the reading of the other multiplied by two.

In any two-phase, three-wire system the power can be measured by two wattmeters connected as in Fig. 166.

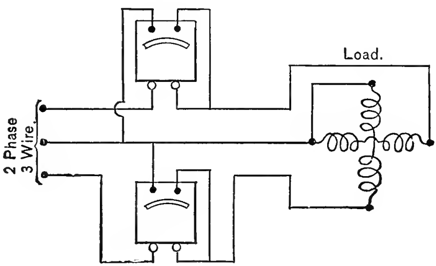


Fig. 166.

The sum of the instrument readings is the whole power. In a two-phase, three-wire system, where all the load is con-

nected between the outside wires and the common wire, and none between the outside wires themselves, and where the load is balanced, then one wattmeter can be used to measure the whole power by connecting its current coil in the common wire nd its pressure coil between the common wire and one outside wire first, then shifting this connection to the other outside wire, as indicated in Fig. 167. The sum of the instrument readings in the two positions is the whole power. A wattmeter made with two pressure coils could have one connected each way, and the instrument would automatically add the readings, giving the whole power directly.

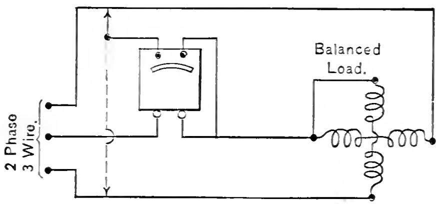


Fig. 167.

Or, again, a high non-reactive resistance could be placed between the two outside wires and the pressure coil of the wattmeter connected between the common wire and the center point of this resistance. This requires that the wattmeter be recalibrated with half of this high resistance in series with its pressure coil.

With the exception of the two-phase systems, the power in any balanced polyphase system may be measured by one wattmeter whose current coil is placed in one wire, and whose pressure coil is connected between that wire and the neutral point. The instrument reading multiplied by the number of phases gives the whole power. The neutral point may be on an extra wire, as in a three-phase, four-wire system; or may be artificially constructed by connecting the ends of equal non-reactive resistances together, and connecting the free ends one to each of the phase wires.

With the exception of the two-phase systems, the power in any n-phase, n-wire system, irrespective of balance, may be determined by the use of $n-1$ wattmeters. The current coils are connected, one each, in $n-1$ of the wires, and the pressure coils have one of their ends connected to the respective phase wires, and their free ends all connected to the nth wire. The *algebraic* sum of the readings is the power in the whole circuit. Depending upon the power factor of the circuit, some of the wattmeters will read negatively, hence care must be taken that all connections are made in the same sense; then those instruments which require that their connections be changed, to make them deflect properly, are the ones to whose readings a negative sign must be affixed.

Some specific connections for indicating wattmeters in three-phase circuits are shown in the following figures. Fig. 168 shows the connection of three wattmeters to meas-

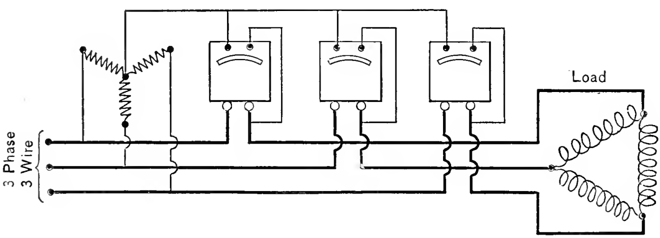


Fig. 168.

ure the power in an unbalanced three-phase system. All the readings will be in the positive direction, and their sum is the total power. If a fourth, or neutral wire be present, it should be used, instead of creating an artificial neutral, as shown. The magnitude of the equal non-reactive resistances, used to secure this neutral point,

must be so chosen that the resistances of the pressure-coils of the wattmeters will be so large, compared thereto, as not to disturb the potential of the artificial neutral point.

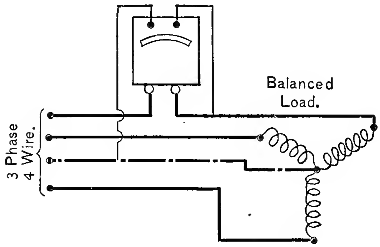


Fig. 169.

Fig. 169 shows the connection of one wattmeter, so as to read one-third of the whole power in a balanced, three-phase, four-wire system. If the system be three-wire, a neutral point may be created as in Fig. 168.

Fig. 170 shows the connections of two wattmeters for the determination of the power in balanced or unbalanced three-phase systems, avoiding the necessity of a neutral point. The algebraic sum of the instrument indications is the whole power. If the power-factor be greater than .5, both instruments will give positive readings; if it be less, one instrument will give a negative reading. With low power-factors, such as given by a partially loaded induction motor, it is sometimes difficult to determine whether the smaller readings are negative or not. If in doubt, give the wattmeters a separate load of lamps (power-factor = 1) and make the connections such that both instruments deflect properly. Then connect them to the load to be measured. If the

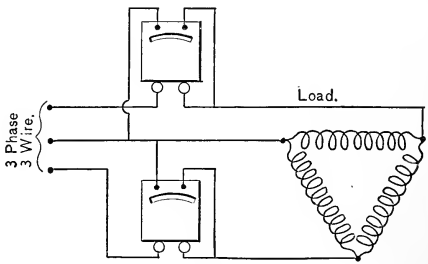


Fig. 170.

terminals of one instrument have to be exchanged, then to the readings of that instrument must be affixed the negative sign. Fig. 171 shows the connections for one wattmeter in a balanced three-phase circuit, independent of a neutral point. The free end of the pressure coil is connected first to one of the wires opposite that in which the current coil is connected, then to the other. The algebraic sum of the readings in the two positions is the total power. Both readings will be positive if the power-factor is greater than .5; but one of them will be negative if it is less. Hence care must be used to avoid confusion of signs at low power-factors. This method, requiring a two-throw switch to change the connection, two readings of the instrument, and, if used on a load varying from high to low power-factor, a commutator, to change the pressure coil connections, has little advantage over the method of Fig. 169, save that it dispenses with the necessity for a neutral point.

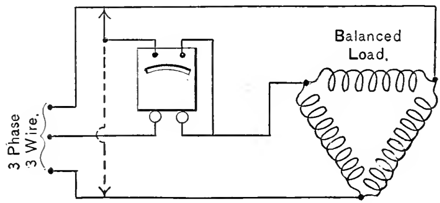


Fig. 171.

Six-phase circuits are used generally only between the step-down transformers of three-phase transmission systems and the alternating-current ends of rotary converters; hence they are always balanced. They can then be measured by the method of Fig. 169, where a neutral is employed, or the three alternate wires may be considered a three-phase system, the method of Fig. 171 employed, and the three-phase power thus determined multiplied by 2 to give the total power. If the circuit should be unbal-

anced, five instruments would be necessary, as stated earlier in this section.

The student is expected to construct circuits according to the various figures just given, and convince himself that the wattmeters do give the true power. If the load be of lamps, the power in each may be measured by a voltmeter and ammeter used at their terminals; then by connecting in star and in delta, balanced and unbalanced, the accuracy of the wattmeter indications can be checked.

In following Fig. 166 or Fig. 167, it should be remembered that a two-phase current cannot be secured from an armature with a mesh winding, such as a rotary converter must have; and that any attempt to make a two-phase, three-wire system out of a quarter-phase system will be disastrous. To get two-phase current from such a machine, the quarter-phase current must be passed through the primaries of two similar transformers, two opposite wires going to one, the other two to the other. The transformer secondaries will then deliver true two-phase current, and the circuits may be united in a three-wire system.

107. Exp. 11. Calculation and Measurement of the Resulting Impedance of a Number of Impedances in Series.— Arrange apparatus as shown in Fig. 172. Determine the impedance Z of the whole circuit from the readings of the voltmeter and ammeter.

$$Z = \frac{E}{I}.$$

Independently determine the ohmic resistance of the circuit with the same current flowing. The magnitude of the current affects the resistance of the lamp.

Solve for the reactance, $X = 2\,\pi f L$, from the equation,

$$Z = \sqrt{R^2 + \omega^2 L^2}.$$

Determine the angle of lag ϕ from

$$\tan\phi = \frac{\text{reactance}}{\text{resistance}}.$$

$$\phi = \tan^{-1}\frac{X}{R}.$$

Determine the reactance, resistance, and impedance of Coil 1, Coil 2, and the lamp, individually. The first two can be derived from the data of Exp. 5 without further measurements.

Graphically determine the total reactance, resistance, impedance, and angle of lag by combining the individual parts in a parallelogram of impedances as described in § 26. A convenient scale for the actual plotting for a drawing-board 25″ × 30″ is 2 ohms = 1 cm.

Make a report in the form of a table such as the following. The variation, with careful work and good instruments, should not exceed 2%.

	Determined		% Variation.
	Graphically.	Experimentally.	
R . . .			
X . . .			
Z . . .			
ϕ . . .			

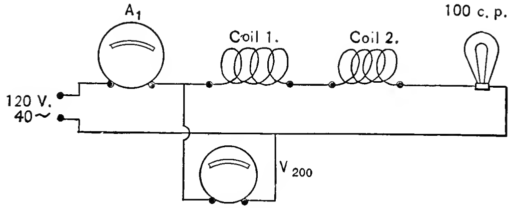


Fig. 172.

108. Exp. 12. Calculation and Measurement of the Resulting Impedance of a Number of Impedances in Parallel. — Use the same impedances as in the last experiment, but arranged as in Fig. 173. As before stated, the voltmeter-ammeter-resistance method of solving inductive circuits is inapplicable to branched circuits; so the wattmeter must be used as shown.

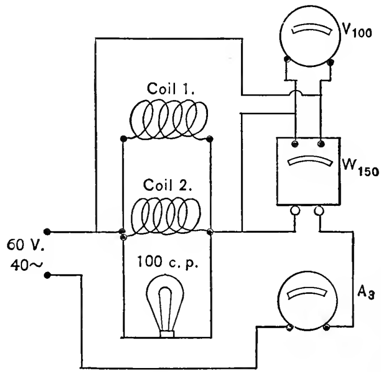


Fig. 173.

Determine the equivalent impedance from

$$Z = \frac{E}{I}.$$

Determine the angle of lag in the main circuit by

$$\cos \phi = \frac{P}{EI}.$$

Determine the equivalent resistance, R (which is not the actual resistance of the parallel arrangement), from

$$R = Z \cos \phi.$$

Determine the equivalent reactance from

$$X = Z \sin \phi.$$

All the constants of Coil 1 and Coil 2 are known; but the resistance of the lamp had better be redetermined for the particular current used in it.

Combine the admittances of these parts of the branched circuit into a polygon of admittances according to § 28. Take the reciprocal of the resulting admittance, — that is, the equivalent impedance, — and resolve it into its component parts of equivalent reactance and equivalent resistance. The actual plotting may be done on 25″ × 30″ drawing-board to the scales 2 ohms = 1 cm. and 1 unit of admittance = 1000 cm.

Make a report in the form of a table such as is used in the last experiment. The variation should not exceed 3%.

109. Exp. 13. Calculation and Measurement of Resulting Impedance of any Series-Parallel or Parallel-Series Arrangement of a Number of Impedances. — Arrange the apparatus as in Fig. 174, or according to any other scheme if it be desired to vary the experiment.

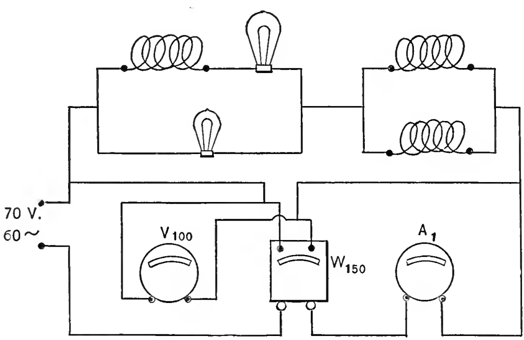


Fig. 174.

Determine the values of the resulting or equivalent R, X, Z, and ϕ, as in Exp. 12.

Also determine the same quantities for the individual parts of the circuit under the conditions of use if they be not already known.

In the graphic determination pursue the following steps : —

1. Find the equivalent impedance of Coil 3, and the 100 C.P. lamp, calling it M.
2. Find the equivalent impedance of the 50 C.P. lamp, and M, calling it N.
3. Find the equivalent impedance of Coil 1, and Coil 2, calling it P.
4. Find the equivalent impedance of P and N. This will be the required impedance of the whole circuit, and should be resolved into its component parts of equivalent resistance and equivalent reactance. Measure ϕ, the angle between the impedance and the resistance.

Make a report in the form of a table as in the two preceding experiments. The variation of the determinations by the two method should not exceed 3%.

110. Exp. 14. Efficiency and Regulation of a Transformer. — Arrange the apparatus as in Fig. 175. A two-throw switch allows the same voltmeter to read either primary or secondary pressure. The ammeter A_3 may be used on the lower readings. The transformer used is the $\frac{1}{2}$ K.W. 1 to 2, stepping up from about 58 volts to 116, its rated range. It is operated at its rated frequency, 60 ~.

Increase the load from 0 to 1 K.W. (100% overload) by suitable steps. At each step take readings of the primary volts, primary watts; secondary volts and secondary am-

peres. Since the load is non-inductive, the product of the secondary volts and amperes gives the secondary watts.

Determine the efficiency and the regulation, both in per cent, for each set of readings from

$$\% \text{ efficiency} = \frac{\text{watts secondary}}{\text{watts primary}} \times 100.$$

$$\% \text{ regulation} = \frac{\tau \text{ volts prim.} - \text{volts sec.}}{\text{full-load sec. volts}} \times 100.$$

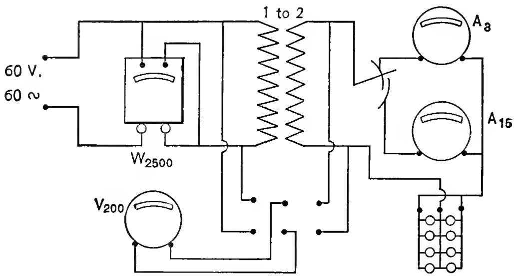


Fig. 175.

Plot two curves on the same sheet, having as their common abscissæ both watts and per cent of full-load secondary, and as their respective ordinates per cent efficiency and 100% — per cent regulation.

111. Exp. 15. Determination of Load Losses in a Transformer. — The core losses are usually considered independent of the load, while all those that vary with the load are called the load losses. Their chief component is, of course, the I^2R loss in the copper, but there may be some eddy current and local hysteresis losses that vary with the load, and a determination of them all is made as follows : —

Arrange the apparatus as in Fig. 176. The 1 to 2

0.5 K. W. transformer is used with its low-tension side short-circuited. There will be but a small pressure generated therein, and its current will demagnetize the core almost entirely; hence all the losses measured may be considered as load, not core losses. Care must, of course, be taken to control the amount of current passing through the transformer.

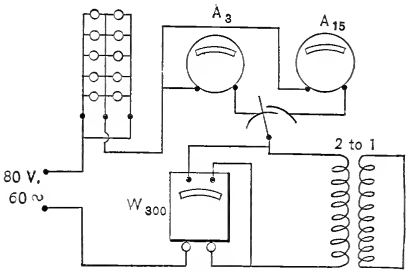


Fig. 176.

Adjust the lamps so that about 100% overload current, 10 amperes, is shown by A_{15}. Read the ammeter and the wattmeter. Reduce the current by a suitable amount, and read again. So continue down to zero amperes, substituting A_3 for A_{15} when the readings on the latter become unsatisfactory.

Plot a curve with load in amperes as abscissæ and load loss in watts as ordinates.

Take care that the wire short-circuiting the low-pressure coil is of low resistance, and has good contacts. Note also that the pressure leads from the wattmeter should go direct to the terminals of the transformer, as, in general, the resistance of the wires leading to it is not negligible in comparison with the resistance of the coil itself.

If the current exceeds the ampere capacity of the wattmeter, it is advisable to put in a single-pole switch to short-circuit the current coil at all times save when a reading is being taken.

112. Exp. 16. Determination of Core Losses of a Transformer, and Construction of an Efficiency Curve. — The core losses, hysteresis chiefly, are constant at all loads. Hence, the energy supplied to a transformer when its secondary is open-circuited, is practically a measure of these losses.

Connect a wattmeter in the primary circuit of the ½ K.W. transformer when its primary is supplied with pressure at its rated voltage and frequency, and its secondary is open-circuited.

The wattmeter reading is the core loss.

From a knowledge of the core loss and the load losses at various loads, construct an efficiency curve for various loads from 0 to 100% overload (secondary), the efficiency in per cent at any load P_1 being

$$\epsilon = \frac{P_1 - (P_l + P_c)}{P_1} \times 100,$$

where P_l and P_c are the load and core losses respectively at the load P_1.

This curve should be similar to the efficiency curve found in Exp. 14.

113. Ex. 17 and 18. Simultaneous Pressure and Current Curves from Primary and Secondary of a Transformer. — It is desired to get these curves for two conditions. First, Exp. 17, with a full non-inductive load, and second, Exp. 18, with an equal (in amperes) very inductive load.

Arrange apparatus as in Fig. 177. For the non-inductive load, lamps are suitable, for the inductive load the primaries of unloaded transformers are good; and to get a nice adjustment Coil 1 can be put into circuit, and

the current in it adjusted by moving its iron core in or out.

It might be here remarked that if the transformer is supplied with current from a rotary converter, the *E.M.F.* balance described in Exp. 2 cannot use direct current from the same source as that which runs the converter, even though they be put on opposite sides of a three-wire system. A separate source of direct *E.M.F.* must in such case be supplied for the balance, either from a separate direct-current generator or from a sufficient number of

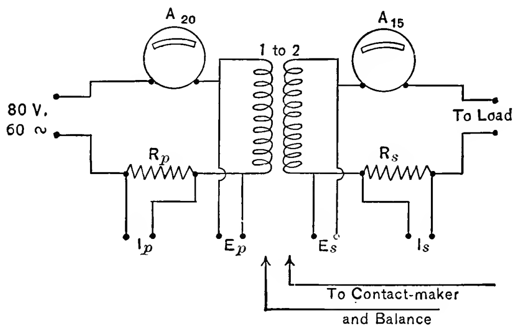


Fig 177.

cells of storage battery. If, however, the alternating current be passed through another transformer before being applied to the one under test, this trouble does not arise; but the introduction of the second transformer has a disturbing effect on the wave-shape.

It will be seen that the apparatus is merely an elaboration of that in Exp. 4, a four-way double-pole switch being used instead of the two-throw switch of the former experiment. This switch is conveniently made of mercury cups in a block of wood. For the $\frac{1}{2}$ K. W. transformer at 58 to 116 volts, the reading on A_{15} should be about 4.4 amperes.

Suitable values for the non-inductive resistances are, $R_p = 2.2$ ohms, $R_s = 4.4$ ohms. These must be able to carry the currents without overheating, and must not be allowed to change their resistance due to change of temperature.

Proceed as directed in Exp. 4, taking readings every twelve electrical degrees throughout half a cycle. Take all four readings before moving the contact-maker.

Plot the four curves of Exp. 17 on one paper, and those of Exp. 18 on another. In each case degrees will be the common abscissæ. Both pressure curves of either experiment must be plotted to one scale of ordinates, both current curves to another. Careful work will show a phase difference slightly less than 180° between the primary and secondary pressures and currents, particularly in Exp. 18.

114. Exp. 19. Calculation and Measurement of the Mutual-inductance of Transformer Coils at No Load. —

(*a*) Measure the self-inductance of the primary and of the secondary coils by the method of Ex. 5, first part, the coil not under test being left open-circuited.

(*b*) Do the same by the method of Exp. 5, second part. The results in the two cases should be alike if attention is paid to the following point. In measuring the primary inductance, apply the rated voltage so that it will send the charging current. In measuring the secondary, adjust the impressed voltage so that just such a current will flow as will give the same ampere-turns in the secondary as there were in the primary when it was being measured. If this precaution be not taken, the results will be changed by the effects of varying load, according to Exp. 7.

(*c*) Using the average of the values of L_p and L_s as

found in (*a*) and (*b*), calculate the value of M on the supposition of no magnetic leakage, from

$$M = \sqrt{L_s L_p}.$$

(*d*) Measure the mutual induction by the method of Exp. 8 *a*, taking care that the ampere turns are the same in each case, and the same as were used in (*a*) and (*b*).

This last result may be slightly less than that arrived at in (*c*) because of magnetic leakage.

Care should be taken throughout that the frequency be kept constant.

115. Exp 20. Practice in Three-Phase Transformer Connections. — Three similar 1 to 1 transformers may be conveniently used for this experiment. The student may have to exercise some ingenuity in determining the direction of winding in the coils. When each of the following connections has been made, excite the primaries by a three-phase current, and measure the pressure between each of the secondary wires, seeing that all three sides have the same and the expected voltage.

(*a*) Connect both primaries and secondaries in Y, as shown in Fig. 88. See that $E_s = E_p$. Then make the secondary a three-phase, four-wire system, and see that the voltage between any outside wire and the middle wire is $\frac{E_p}{\sqrt{3}}$.

(*b*) Connect both primaries and secondaries in Δ, Fig. 87, and see that $E_s = E_p$. Disconnect one transformer from the circuits, and observe that the three-phase pressure is still maintained in the secondary.

(*c*) Connect the primaries in Y, and the secondaries in Δ, as in Fig. 90. Observe that $E_s = \frac{E_p}{\sqrt{3}}$.

(*d*) Connect the primaries in Δ, and the secondaries in Y, as in Fig. 89, with a four-wire secondary system. Observe that, with reference to the outside wires, $E_s = \sqrt{3}\ E_p$, while considering any outside wire and the middle wire, $E_s = E_p$.

116. Exp. 21. External Characteristic of an Alternator. — Run the alternator at normal speed and field excitation, both being kept constant during the experiment. Arrange a variable non-inductive load — lamps preferably — so that readings can be taken from 0 load to 50% overload at suitable intervals. At each step note the armature current and the terminal pressure.

Plot a curve with currents as abscissæ, and pressures as ordinates.

117. Exp. 22. Field Compounding Curve of an Alternator. — Run the alternator at constant rated speed. Arrange a variable non-inductive load of lamps, ranging by suitable steps from 0 load to 50% overload; at each step adjust the field current, so that the rated terminal voltage is maintained. Take simultaneous readings of field-current and armature current.

Plot a curve with armature currents as ordinates, and field currents as abscissæ.

Note that the speed must be kept constant, that the terminal pressure must be kept constant, and that readings should be taken only with ascending values of field currents, as magnetic retentivity will distort the curve somewhat if the field current is run too high, and then brought down to the required point.

118. Exp. 23. No-Load Saturation Curve of an Alternator. — Run the alternator at constant rated speed, and

excite the fields from zero up to full excitation, taking, at suitable intervals, readings of the field current, and the no-load armature voltage. Repeat, carrying the excitation from full excitation down to zero.

Plot the two curves on one sheet, using field currents as abscissæ, and terminal pressures as ordinates. The two curves will not exactly coincide, because of the magnetic retentivity of the iron.

Care must be taken always to adjust the field current by increasing from a lower value to a higher when taking the ascending curve; and by decreasing from a higher value to a lower when taking the descending curve.

119. Exp. 24. Full-Load Saturation Curve of an Alternator. — Arrange apparatus as in Fig. 178. The alternator is a 1 K. W. 80 volt, single-phase machine. The machine is given a non-inductive load of lamps, and a heavy current rheostat which has zero resistance on the

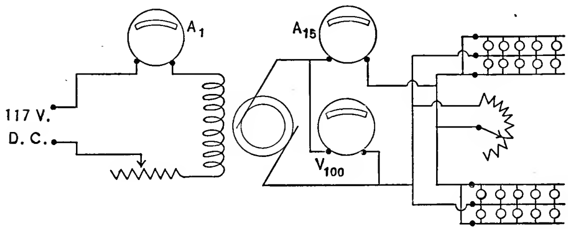


Fig. 178.

last point. Run the alternator at its rated speed. Make the resistance of the external circuit zero — i.e., short-circuit it through the ammeter. Adjust the field rheostat to its maximum resistance, and close the field switch. Increase the excitation by manipulating the field rheostat until the rated full-load current is flowing in the external circuit, as

shown on A_{15}. Take readings of the field amperes, and the terminal volts, the latter being zero at this step. Increase the resistance of the armature circuit by a suitable amount, and readjust the excitation till the rated full-load current is again flowing in the armature, and take readings of field current and terminal voltage. Repeat at suitable steps until full field excitation is obtained.

Plot a curve on the same paper, and to the same scale as that of Exp. 23, using field currents as abscissæ, and terminal volts as ordinates.

Take heed that readings are always taken with ascending values of field current, and when the ammeter in the armature circuit shows rated full-load current. The speed must be kept constant.

120. Exp. 25. Synchronous Impedance of an Alternator. — As stated in § 38, the synchronous impedance of an alternator varies somewhat with the load, but is practically constant at all excitations; hence its determination is easily accomplished in the following manner: —

Arrange the apparatus as shown in Fig. 179. Run the alternator at its rated speed. By means of the field rheostat cut the excitation down to a minimum. Short-circuit the armature through an ammeter and switch as shown. Adjust excitation so that

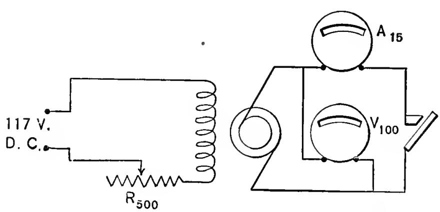


Fig. 179.

the ammeter shows about $\frac{1}{5}$ full-load current, and note the ammeter reading. Open the switch in the armature cir-

cuit and note the terminal volts. Close the switch, readjust excitation so that the load is increased by a suitable amount, and repeat the readings. Repeat until a limit is reached, either because full field excitation has been obtained, or because the machine is being too severely overloaded. Which of these two conditions arises first depends upon the synchronous impedance of the machine.

Calculate the synchronous impedance for each set of readings from

$$\text{Syn. Imp.} = \frac{\text{open circuit voltage}}{\text{short circuit current}},$$

when the readings are for the same excitation and speed.

Plot a curve with armature currents as abscissæ, and the values of the synchronous impedance as ordinates.

Particular attention should be paid that the speed be kept constant, as it is liable to rise on throwing off the load.

121. Exp. 26. Core loss of an Alternator. — The core loss of any armature is determined by measuring the difference in power required to run it with and without field excitation. With an alternator this is most easily done by running the armature by a rated motor and observing the power input thereto. It is desirable to have the quantity sought as large as possible in comparison with the quantities observed; hence the rated motor used should be as small as is practicable.

The alternator must be driven at its rated speed, and the pulleys so proportioned that the motor will run at its rated speed also; or else a special efficiency curve of the motor must be obtained for the speed at which it will be required to run.

A wattmeter placed in the motor circuit will indicate the power input thereto; or, if it be a direct-current motor, a voltmeter and ammeter can be used.

Let A = watts input to motor when the alternator field is not excited.
and m = efficiency of motor at this input.
Let B = watts input to motor when the alternator fields are fully excited,
and n = efficiency of motor at this input.

Then the core loss in watts is

$$P_c = Bn - Am.$$

It is well to repeat the measurements a number of times and average the results.

Since the losses in shafting and belting are practically the same at all loads, these do not affect the accuracy of the results.

122. Exp. 27. Complete Test of a 1-H.P. Three-Phaso Induction Motor.—As a test of the motor performance solely, the voltage at the motor terminals should be kept constant throughout the test. This may easily be accomplished if the motor is run from a separate alternator. If, however, it is run from an inverted converter, and particularly if the desired voltage has to be obtained by transformation, there will be a slight drop of voltage as the load increases.

Since the power-factor in this test will run from very low to about 80%, the method of measuring three-phase power shown in Fig. 169 will be used, as it requires but one instrument reading, and leads to no uncertainty as to

algebraic signs. The apparatus used is simply an ammeter, a voltmeter, and a wattmeter connected into the motor circuit. Fig. 180 shows the arrangement, the two wattmeters not being used at once, but being alternative,— one for high readings, the other for low,—thus securing a greater accuracy over a wide range. The motor when stalled takes about 18 amperes; so this is its momentary

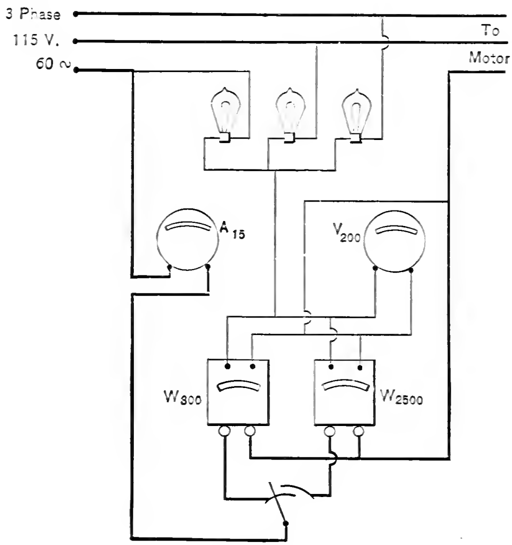


Fig. 180.

starting current. Care must be taken in starting up that the measuring instruments are not injured by such a flow of current. The larger wattmeter has a capacity of 2.5 K. W., and a 25-ampere limit; the smaller a capacity for 300 watts, and a 5-ampere limit. Either of these, as well as the voltmeter, will stand the rated motor pressure, — 110 volts.

The power output of the motor is absorbed in a strap

brake, as shown in Fig. 181. With a 4.5″ pulley at 1800 revolutions the spring balances should have ranges of about 30 lbs. and 4 lbs. respectively.

The motor must be supplied with current at its rated voltage and frequency, and the frequency must be kept constant throughout the experiment.

Observations.— Take readings at suitable intervals, — say steps of 4 lbs. each on the larger scale, — from no load to the stalling of the motor. Do not leave the motor stalled, as it overloads the instruments.

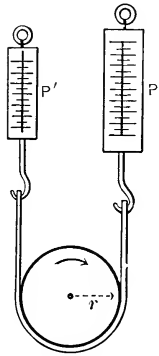


Fig. 181.

At each step take readings of the wattmeter, ammeter, voltmeter, both spring balances, and the speed of the motor.

Repeat the experiment three times with fifteen-minute intervals between the repetitions. The scale readings, P, will be the same, at any one step, for all three trials, and the other values can be averaged to partially eliminate errors of observation.

Calculations. — Using the average values of the three readings at any one step, fill out the following table : —

1	2	3	4	5	6	7	8	9
Watts Output.	Watts Input.	Volts at Terminals	Amperes per Phase	Volt-Amperes Input.	Power-Factor %	Efficiency %.	Apparent Efficiency %	Slip %.

$$(1) \text{ Watts output} = \frac{\frac{d}{12} V\pi (P - P')}{33{,}000} 746,$$

where

d = diameter of pulley in inches.
V = revolutions per minute.
$(P - P')$ = difference in scale readings in pounds.

(2) Watts input = 3 × wattmeter reading.

(3) Volts at terminals = voltmeter reading.

(4) Amperes per phase = ammeter reading.

(5) Volt-amperes input $= \sqrt{3} \times$ volts at terminals × amperes per phase.

$$(6) \text{ Power-factor } \% = \frac{\text{Watts input}}{\text{Volt-amperes input}} \times 100.$$

$$(7) \text{ Efficiency } \% = \frac{\text{Watts output}}{\text{Watts input}} \times 100.$$

$$(8) \text{ Apparent efficiency} = \frac{\text{Watts output}}{\text{Volt-amperes input}} \times 100.$$

$$(9) \text{ Slip } \% = \frac{\frac{60f}{p} - V}{\frac{60f}{p}},$$

where

V = revolutions per minute,
f = frequency,
p = number of pairs of poles.

Plotting of Curves. — Plot eight curves on one paper. All the curves will have watts output as abscissæ. The points of 25%, 50%, 75%, 100%, and 125% of full load should also be indicated.

The ordinates for the first seven curves are taken from columns 2 to 8 respectively.

The ordinates for the last curve are found by subtracting the per cent slip from 100%.

Curves should be marked with the names appearing at the heads of the columns from which their ordinates were taken. The curves from columns 2 and 5 should be to the same scale of ordinates. Those from columns 6, 7, 8, and 9 will all have the same scale of ordinates, which will be per cents, and should run from 0 to 100%.

There will thus be four scales of ordinates, and they should be marked respectively, "Watts or Volt Amperes," "Volts," "Amperes," and "Per Cent." On the margin state the name and size of the machine, and the date of test.

123. Exp. 28. Complete Test of a ½ H.P., Three-Phase Induction Motor, run on a Single-phase Circuit Through a Condenser-Compensator. — The function of the condenser-compensator was discussed in § 67. The arrangement of apparatus is shown in Fig. 182. Another wattmeter and

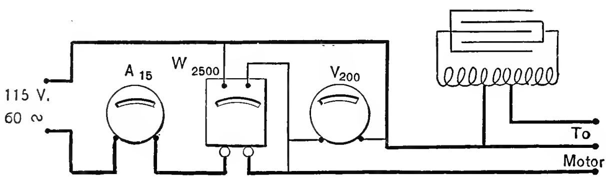


Fig. 182.

another ammeter may be used to alternate with those shown to secure greater accuracy in the lower ranges if considered advisable.

The same absorption dynamometer is used as in Exp. 27; and the directions there given for taking observations, and for calculating and plotting results, should be followed with the following exceptions: take readings at 2-lb. steps,

since the motor is half the size of the other; since this is single-phase, the wattmeter readings go direct in column 2, and the products of the volts by the amperes go direct in column 5.

It may be found that the capacity of the condenser-compensator has been so proportioned that in plotting the results, curves 2 and 5, and also 7 and 8, will be nearly coincident, and that curve 6 is practically a straight horizontal line.

124. Exp. 29. Methods of Synchronizing. — Synchronous motors and also converters must be synchronized before being connected to the mains from which they receive their power. There are a number of ways of doing this, of which the best depends upon attendant circumstances. (*a*) The motor and generator may be electrically connected while at rest, and the latter started up slowly, the motor — not loaded — then starting up and running synchronously. (*b*) The field circuit of the motor may be left open, and the armature started up — without load — as an induction motor until near synchronism, and the field switch then closed. In large machines this endangers the insulation of the field coils. (*c*) The armature may be brought to speed mechanically, either by a small direct connected induction motor or by a belt from some moving pulley. (*d*) In converters the machine can be started and brought to speed from the direct-current end like a direct-current motor, if there be direct current available. This requires a starting-box and a field rheostat.

The two convenient methods for synchronizing the 1 K.W. three-phase converter are (*b*) and (*d*); the former

will be practiced in Exp. 30, the latter is the subject for the present experiment.

Arrange the apparatus as shown in Fig. 183. At starting, the field coils of the converter must be excited from the source of direct current, but when running as a converter the machine must be excited from its own brushes. This necessitates the two switches, *a* and *b*. These switches must not be both open at once, at least while the machine is running from the direct-current end; and if they are not rightly connected the direct-current

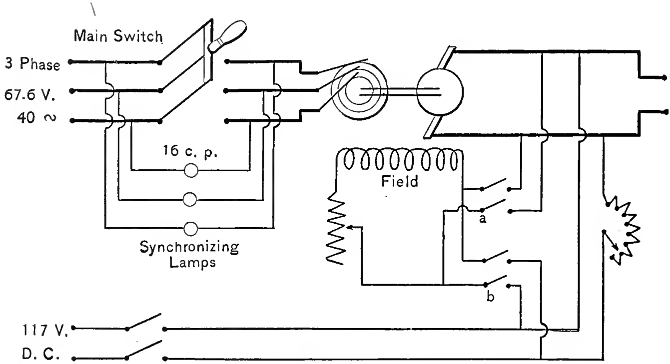


Fig. 183.

source will be short-circuited when they are both closed at once. It is best, after the set-up is made, to test across the switches with a voltmeter. The switch must not be closed if any pressure shows across the gap it is intended closing.

When the connections have been properly made, open the main switch and switch *a*, close switch *b* and the switch from the direct-current source of supply, and start the machine up as a direct-current motor. When the starting-box is completely on, first close *a*, then open *b*.

Then manipulate the field rheostat until the machine reaches synchronism. The synchronizing lamps will all be dark at once when the machine is in step. When the periods of darkness become quite long, say several seconds, the main switch can be closed, the switch from the direct-current source be opened, and the machine will be running as a self-excited converter.

If all the lamps do not get dark at once, but two stay lighted while the other is dark, the generator currents are in such directions as to tend to reverse the direction of rotation of the converter armature. Two of the leads in the alternating-current circuit should then be transposed. It might here be noticed that if an inverted converter be used as a source of alternating currents, and it be desired to synchronize another converter with it as described above, and if an Edison three-wire system be the common source of direct-current supply to these machines, then care must be taken that both converters are connected to the same side of the system. If they be connected to opposite sides, then when the machines are in step there is a pressure of 117 volts across the main switch, and closing the latter would naturally cause the blowing of some fuse.

125. Exp. 30. Variation of Lag or Lead of Current in a Three-Phase 1 K.W. Synchronous Motor. — Arrange the apparatus as in Fig. 184. Either the 3 or the 15 ampere ammeter can be put in circuit, according to the load. Remember that in starting up there will be an excessive flow of current.

The direct-current brushes on the machine may be removed for this experiment. Synchronize the motor by

letting it run as an induction motor till near synchronism. Then close the field switch, having first adjusted the rheostat, so that about normal field current will flow. The machine will fail to go into step if this adjustment is not made. Premature closing of the field switch is also a cause of failure to synchronize. It is easy to tell if the machine goes into step or not. If it does, the current in the armature circuit goes down; if it does not, the machine slows down and even stops, and the current goes up.

(*a*) When the motor is synchronized, reduce the field current as far as possible, without losing step. In some

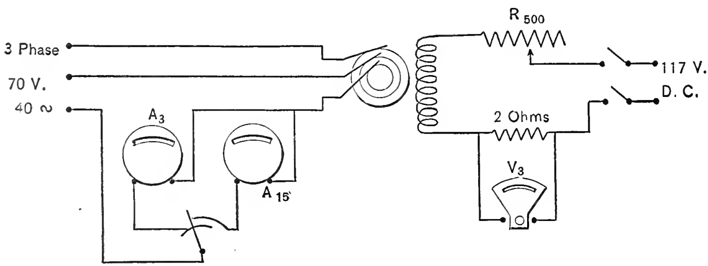


Fig. 184.

machines it may be reduced to zero, the residual magnetism affording enough field to keep the armature synchronized. From this point increase the field current by suitable steps to the maximum allowable current, or until the machine loses synchronism. At each step note the field current and the armature current. Then from the maximum field current, decrease to the minimum by the same steps, taking readings again of the two ammeters. Due to magnetic retentivity the two curves will not coincide.

Plot the two curves on one paper, using field currents as abscissæ, and armature currents as ordinates.

That excitation at which the no-load armature current is a minimum, is called the normal excitation of a synchronous machine.

(*b*) Repeat the foregoing, save that a strap-brake load is applied to the motor. This load should be adjusted to about 75% of full load, and left constant. It will be found that the motor will not submit to so wide a range of field currents when loaded as when running light.

Plot these curves on the same sheet.

126. Exp. 31. Commercial Efficiency of a Synchronous Motor. — The same arrangement of the same apparatus is here used as in Exp. 27, save that it is applied to a three-phase synchronous motor, whose field coils must be separately excited from a direct-current source, and with a suitable rheostat in series.

Synchronize the motor by the method of Exp. 30, being careful that the excessive starting current does not pass through the coil of the low-reading wattmeter. When the armature is in step, adjust the field rheostat to give normal excitation; i.e., so that the armature current is a minimum. This adjustment must not be changed during the experiment. The frequency should be kept constant, and the voltage also if possible; if not, account should be taken of its fall, and a voltage curve drawn on the same sheet as the efficiency curve.

Load the motor by the strap-brake shown in Exp. 27, increasing by suitable steps from zero till the motor falls out of step. At each step read the wattmeter and the two spring balances. Repeat the experiment three times, each time stopping at the same points on the larger spring

balance, so that the other values can be averaged, reducing errors of observation.

Plot a curve with watts output and per cent of load as abscissæ, and per cent efficiency as ordinates.

127. Exp. 32. Curves of Current and Power-Factor of a Synchronous Motor, with (*a*) **Super-Excitation, and** (*b*) **Sub-Excitation.** — The arrangement of apparatus is that of Exp. 27, applied to the synchronous motor.

For the first part of the experiment, the fields should be excited by a current about 50% greater than the normal field current, and for the second part by a current about 50% less. The frequency and the voltage should be kept constant.

For each part load the motor with the strap-brake, increasing from zero by suitable steps till the armature falls out of synchronism. At each step take readings of the ammeter, voltmeter, and wattmeter, as well as of the two spring balances. Repeat each part three times, averaging the results.

Tabulate the results under columns headed "Watts Output," "Watts Input," "Volt-amperes Input," and "Power-factor."

Plot on one paper the curves for the super-excited condition, making watts output the common abscissæ, and armature currents and power-factors respectively the ordinates.

Plot on another sheet similar curves for the sub-excited condition.

128. Exp. 33. External Characteristic of a Converter, A.C. to D.C. with Self-Excitation. — Arrange apparatus as in Fig. 183, Exp. 29. When the converter is running

from the alternating end, and free from the source of direct-current supply, adjust the field rheostat to that point that gives a minimum armature current at no load. If this point is not already known, it will be necessary to put the 15-ampere ammeter in one of the alternating-current mains to determine it.

When the above conditions are fulfilled, load the direct-current end of the converter with lamps, from 0 up to 50% overload (say 15 amperes), by steps of about one ampere each. At each step take readings of the armature current and the terminal pressure, using the standard direct-current instruments for the purpose.

Plot a curve with armature currents as abscissæ, and terminal pressure as ordinates.

129. Exp. 34. Efficiency of a Converter from A.C. to D.C. — The arrangement of apparatus is that of the last experiment with the addition of a wattmeter suitably connected in the alternating-current mains to measure the power input. In fact, all the necessary data for Exp. 33 are incidentally secured in the course of this experiment.

Run and excite converter as in the last experiment. The frequency and the voltage should be kept constant. The direct-current end is to be loaded with lamps by suitable steps from 0 to 50% overload. The watts input can be determined from the wattmeter reading, the watts output from the product of the voltmeter and the ammeter reading.

Plot an efficiency curve.

Note. — The brush friction of the direct-current brushes may be so great that the converter cannot be synchronized by the method of starting as an induction motor, the slip being so great as to prevent its picking up

when the field circuit is closed. In such case either the direct-current brushes must be temporarily removed, or the machine must be synchronized by some of the other methods given in Exp. 29. If the converter hunts so badly as to interfere with the instrument readings, it may be because the direct-current brushes are not in proper adjustment, and the selection of a better commutating plane will remedy the trouble.

130. Exp. 35. Efficiency of an Inverted Converter. — Since it is inconvenient to put a variable, non-inductive, balanced load on a three-phase circuit, the single-phase rings will be used in this experiment.

The arrangement of apparatus requires a direct-current ammeter and voltmeter on the *D.C.* end, and a wattmeter on the *A.C.* end. Or two wattmeters can be used if available. The converter is started from the *D.C.* end by means of a starting-box, the field rheostat is adjusted until the armature is running at its rated speed. This speed must be kept constant during the test by manipulating the rheostat. A non-inductive load is applied to the *A.C.* end, increasing by suitable steps from 0 to 50% overload.

Plot an efficiency curve and an external characteristic curve. The latter will approximate a straight horizontal line, being much better than that secured in Exp. 33, because, in this case, the field current is unaffected by any drop of voltage in the armature.

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